Physics 20

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Module 5
CIRCULAR MOTION

Rocky View School Division
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Edmonton Public Schools
Calgary Board of Education
Alberta Education

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CIRCULAR MOTION

Red Deer Catholic Regional Schools

EDMONTON PUBLIC SCHOOLS

Calgary Board of Education

Alberta Education

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- Black Gold Regional Schools
- Calgary Board of Education
- Edmonton School District No. 7
- Peace Wapiti School Division No. 76
- Pembina Hills Regional Division No. 7
- Red Deer Catholic Regional Division
- Rocky View School Division No. 41

Physics 20
Module 5: Circular Motion
Student Module Booklet
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Education, **http://www.education.gov.ab.ca**
- Learning Resources Centre, **http://www.lrc.education.gov.ab.ca**
- Tools4Teachers, **http://www.tools4teachers.ca**

Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Contents

Unit C Introduction

This unit will extend your study of kinematics and dynamics to uniform circular motion, mechanical energy, work, and power. This will prepare you for further study of circular motion, conservation laws, and moving particles in magnetic fields in Physics 30.

The major concepts developed in this unit will allow you to

- explain circular motion using Newton's laws of motion
- explain that work is a transfer of energy and that conservation of energy in an isolated system is a fundamental physical concept

Think about the following questions as you complete this module:

- What conditions are necessary to maintain circular motion?
- How does an understanding of conservation laws contribute to an understanding of the universe?
- How can mechanical energy be transferred and transformed?

Unit Assignment

You are required to complete a multi-part question in the Unit C Assessment. This is the type of question you would see on the Physics 30 Diploma Exam.

Your teacher may assign additional or alternate assessment items for this unit.

Concept Organizer

Unit C – Circular Motion, Work, and Energy

Module 5: Circular Motion

Lesson 1 – Understanding Circular Motion
Lesson 2 – Newton's Laws Applied to Vertical Ciruclar Motion
Lesson 3 – Kepler's Three Laws of Planetary Motion
Lesson 4 – Planetary and Satellite Motion

Module 6: Work and Energy

Lesson 1 – Work, Potential, and Kinetic Energy
Lesson 2 – Conservation and Mechanical Energy
Lesson 3 - Mechanical Energy and the Work-Energy Theorem
Lesson 4 – Work and Power

Module Descriptions

Module 5: Circular Motion

In this module you will apply your knowledge of Newton's laws to help you understand uniform circular motion in everyday situations. You will use simulations and labs to examine planetary and satellite motion. You will also learn how Kepler's laws were used in the development of Newton's universal law of gravitation.

The essential question that you will be looking at in this module is the following:

- What conditions are necessary to maintain circular motion?

Module 6: Work and Energy

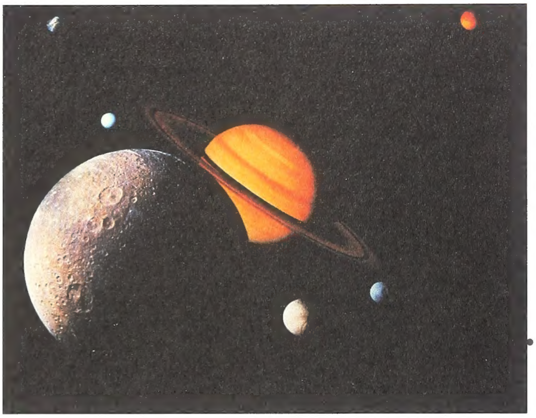
In this module you will study work and its relationship to different forms of mechanical energies. You will also explore the connection between work and power. You will analyze kinematics and dynamics problems that relate to the conservation of energy in an isolated system, and you will examine the change in mechanical energy in a system that is not isolated.

The essential questions that you will be looking at in this module are the following:

- How does an understanding of conservation laws contribute to an understanding of the universe?
- How can mechanical energy be transferred and transformed?

Module 5—Circular Motion

Module Introduction



Digital Vision/Getty Images

In this module you will apply the knowledge of Newton's laws of motion you gained in Unit B to help you understand uniform circular motion in everyday situations. You will use simulations and labs to examine planetary and satellite motion. You will also learn how Kepler's laws were used in the development of Newton's universal law of gravitation.

You will look at the following essential question in this module:

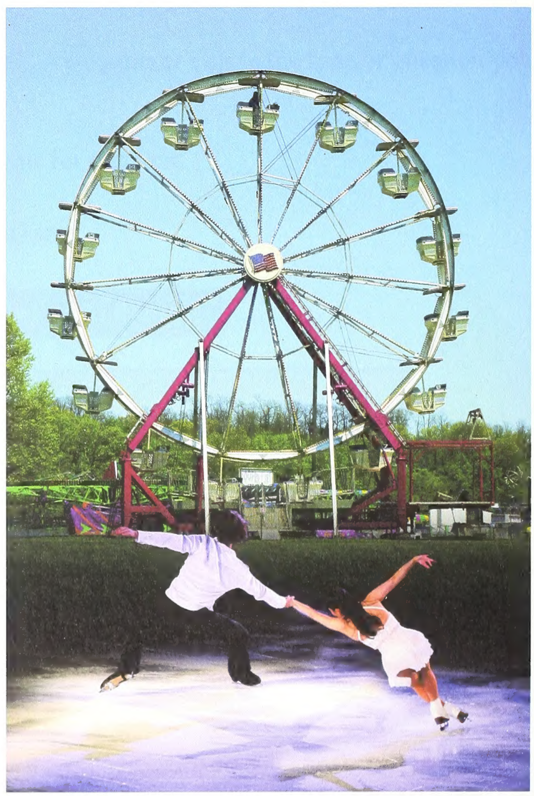
- What conditions are necessary to maintain circular motion?

In preparation for this module's assessment, go to the In This Module section and read the information about the Module 5 Project. You will begin this inquiry project once you have completed all of the lessons in Module 5. This information is also available in the Module Summary section.

Big Picture

How exciting would it be to watch figure skating if it all occurred in a straight line? It might still be fast with jumps, spills, grace, elegance, and showmanship; but, without the spins, it wouldn't be nearly as exhilarating. How exciting would it be to try a carnival ride that had no twists, no spinning, and no turns? The ride might go fast; but if it could only move in a straight line, it wouldn't be much fun. What is it about spinning, rotating, or orbiting that makes these experiences more exciting?

As a student of physics, you know that this excitement has to be related to accelerations and forces and other physics concepts. You might even correctly guess that circles come into play. Circles can be small, like a skater doing a scratch spin; circles can be large, like a Ferris wheel or other carnival ride; circles can even be galactic in scale.



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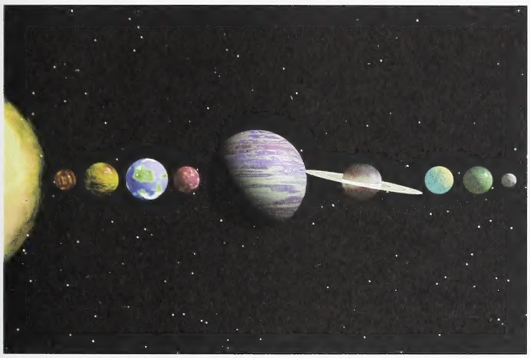
You can see circular motions almost anywhere you look. Each of the pictures you see here shows an example of circular motion on one scale or another with varying forces and accelerations. Circular motion can occur here on the surface of Earth with Earth's gravity being an important force. Other times, the circles are far away from Earth and Earth's gravity plays a much smaller role.



How important do you think Earth is in shaping the orbits of the other planets in the solar system?

The planets, or wanderers of the night sky, intrigued countless generations of astronomers. For a long time, the seemingly inexplicable movements of the planets led to more and more complicated descriptions of their motions. While Galileo expanded people's understanding of astronomy with the invention of the telescope and while Johannes Kepler formulated laws of planetary motion, you will come back to Isaac Newton's work to see the first modern explanation of the rules that determined each planet's place in the sky and the motions of all celestial bodies. The advancement of astronomical studies has been the result of work by a number of scientists—each scientist expanded on the work of a predecessor until arriving at the understandings of today.

In this module you will extend your understanding of Newton's laws into the realm of circular motion and apply free-body diagrams to help you understand the forces involved in circular motion. You will also see, through a historical perspective, how important careful observations and good data are in the development of an understanding of the universe.

As you work through this module, keep the following questions in mind. They will help you understand the role of acceleration in making the universe, and a lot of things in it, go round and round.

- How do Newton's first and second laws apply to circular motion?
- What factors affect acceleration during circular motion?
- How do Newton's laws explain the sensations during vertical circular motion, such as during a loop in a roller coaster?
- What forces come into play during vertical circular motion?
- How did Kepler's laws help calculate the position of the planets as they orbited the Sun?
- How are planetary and satellite motion explained using universal gravitation and the principles of circular motion?

In This Module

Lesson 1—Understanding Circular Motion

A simulation will help you visualize circular motion and examine the application of Newton's first and second laws to circular motion. What is the role of centripetal force in circular motion? What is the relationship between centripetal force and radius, mass, and velocity?

As you work through this lesson, you will explore the following questions:

- How do Newton's first and second laws apply to circular motion?
- What factors affect acceleration during circular motion?

Lesson 2—Newton's Laws Applied to Vertical Circular Motion

This lesson will continue to apply the principles of Newton's first and second laws to vertical circular motion. You will use a simulation that utilizes free-body analysis and Newton's second law to help you understand the acceleration acting on an object that moves along a vertical circular path. What is the difference between the acceleration of an object at the top of its vertical circular path and the acceleration at the bottom of its vertical circular path?

You will explore the following essential questions:

- How do Newton's laws explain the sensations during vertical circular motion, such as in a loop on a roller coaster?
- What forces come into play during vertical circular motion?

Lesson 3—Kepler's Three Laws of Planetary Motion

An examination of the three laws of planetary motion that were formulated by Kepler in the early 1600s forms the basis of this lesson. The focus of these laws is to explain the nature of planetary motion, not to explain why this motion occurs. A simulation, which uses astronomical units, will be used to explore Kepler's three laws and how they accurately describe planetary motion. Exactly how did people come to understand the motion of our solar system in the 1600s?

You will explore the following essential question:

- How did Kepler's laws help calculate the position of the planets as they orbited the Sun?

Lesson 4—Planetary and Satellite Motion

In this lesson you will extend your understanding of planetary motion and look at why planetary motion occurs. What is the relationship between Newton's universal law of gravitation and the orbit of any satellite or moon? How do you explain planetary motion and satellite motion using universal gravitation and the principles of circular motion?

You will explore the following essential question:

- How are planetary and satellite motion explained using universal gravitation and the principles of circular motion?

Module 5 Assessment

The assessment for Module 5 consists of four (4) assignments, as well as a final module inquiry project.

- Module 5: Lesson 1 Assignment
- Module 5: Lesson 2 Assignment
- Module 5: Lesson 3 Assignment
- Module 5: Lesson 4 Assignment
- Module 5 Project

Module 5 Project: Global Positioning Satellites

Getting Started

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This module project is based on an understanding of global positioning systems (GPS) and how they relate to circular motion, gravitational fields, and the past and future impacts of technology on society.

Global positioning systems are proliferating in our society in many ways. Cell phones, cars, pet tags, and other applications are evolving to incorporate this technology.

Applications, such as "arrive alive," will allow cell-phone position, movements, and whereabouts to be tracked in real time. Imagine parents monitoring where their children are at all times, being notified if the phone travels outside a "boundary" or if the motion of the phone is in a vehicle that exceeds the local speed limit. These are issues that are being discussed now, and they are all related to satellite technology that is based on circular motion and gravitational fields. In fact, if you have a cell phone, you are already carrying this technology.

Project Tasks

This inquiry project involves four major questions with two related tasks in each.

1. How is GPS technology used to solve problems?

    a. Investigate where GPS applications are being used, noting societal issues that have "popped-up" as a result of their applications.

    b. Create a list of applications or problems that GPS technology could be used to solve.

2. How does GPS work?

    a. Research the network of satellites that are used to power the current world GPS system, and describe how this technology functions.

b. Report on the orbital specifications of the GPS satellites. For example, report about the mass, altitude, and orbital period of GPS satellites.

3. Describe how principles of circular motion determine the orbital path of a satellite.

    a. Given a satellite's mass and orbital altitude, how is the speed of the satellite determined?

    b. How is a satellite's speed and altitude related to the launch vehicle requirements?

4. How does Earth's gravitational field cause the orbital path of the satellite?

    a. Determine the acceleration due to gravity at the GPS satellite's orbital altitude using Newton's universal law of gravitation.

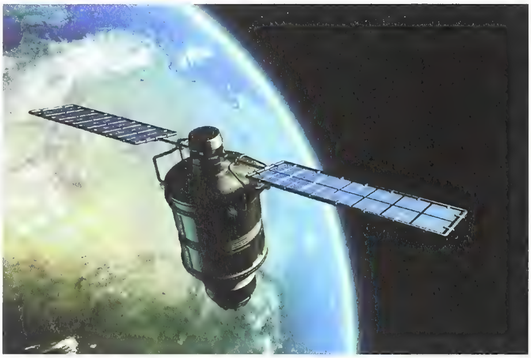
    b. Relate the orbital acceleration to the velocity as a verification using $F_{\text{inward}} = F_g$.

Submitting Your Project

Your project must be submitted as a computer presentation using the software of your choice (e.g., PowerPoint, Word, HTML, PDF).

Your presentation will include a simulation of the GPS orbital path that verifies your calculations. You may use the Weight and Orbits simulation used in Lesson 4, or you may create this simulation using other technologies.

Your presentation will include answers to the four major questions posed under Project Tasks.



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Your presentation should conclude by explaining how orbital knowledge supports the infrastructure for new technologies and their application in solving both old and new problems.

Be sure to include a reference page listing all of your research sources for this project. List all web pages, books, and magazine articles you use.

Research

Begin your research by searching the Internet for the phrase "Global Positioning System GPS."

You might also want to search for a tutorial on GPS.

How Will Your Project be Assessed?

You will be marked according to the following guidelines. You will be graded for your work answering the four major questions listed under Project Tasks.

1. How is GPS technology used to solve problems?			
1: Poor	**2: Satisfactory**	**3: Good**	**4: Excellent**
The project suggests applications that are unrelated to the technology or are not new.	The project suggests only a few limited applications, some not new.	The project explains multiple applications of GPS and suggests new and unique applications.	The project applies GPS to new applications and begins to identify risks and benefits.

2. How does GPS work?			
1: Poor	**2: Satisfactory**	**3: Good**	**4: Excellent**
The project demonstrates a lack of research and understanding of the technology.	The project demonstrates a basic understanding of the technology with some research.	The project demonstrates a solid understanding of the technology with relevant research.	The project demonstrates a solid understanding of the technology with exemplary research.

3. Describe how principles of circular motion determine the orbital path of a satellite.			
1: Poor	**2: Satisfactory**	**3: Good**	**4: Excellent**
The project does not apply relevant physics concepts.	The project applies basic equations to describe one aspect of circular motion.	The project applies all equations with some errors and misconceptions.	The project applies all equations to completely describe the motion of a satellite.

4. How does Earth's gravitational field cause the orbital path of the satellite?			
1: Poor	**2: Satisfactory**	**3: Good**	**4: Excellent**
The project does not apply Newton's laws to the motion.	The project applies one of Newton's laws for a basic explanation of the orbit.	The project applies all of Newton's laws correctly to explain the motion of the satellite.	The project applies and shows manipulation and substitution of Newton's laws to explain the motion of the satellite.

You will also be graded for the presentation and delivery of your Module 5 Project.

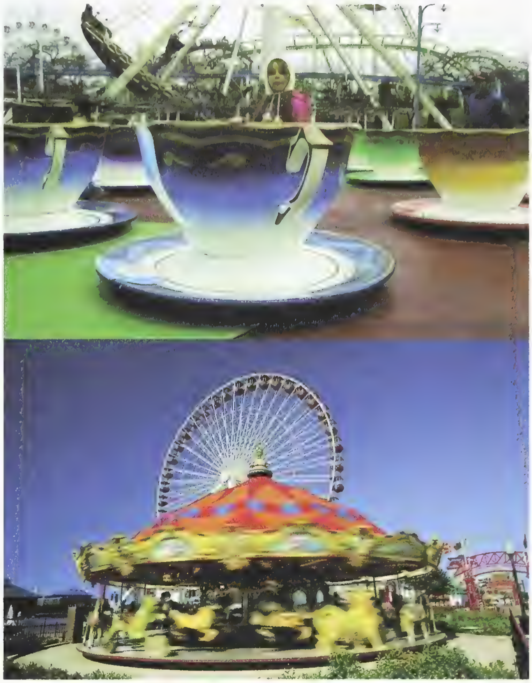
1: Poor	2: Satisfactory	3: Good	4: Excellent
The project presents content in a text-based document only.	The project presents content in multimedia form with limited discussion and explanation.	The project presents content in multimedia form with discussion and use of a simulation.	The project presents content in multimedia form with discussions, supporting documents, a simulation, and suggestions for further study and application.

Total available marks = 20 marks x 3 (weighting) = 60 marks

Lesson 1—Understanding Circular Motion

Get Focused

Going in circles, it seems, is fun. Consider all the rides you find at a local carnival or fair. Nearly every ride involves circular motion. Some rides move in the horizontal plane, such as the teacups ride, which is pictured here; others move in a vertical plane, such as the Ferris wheel and the Tidal Wave (pictured behind the teacups). Other rides move with a combination of vertical and horizontal motions, such as the roller coaster.



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bottom: Photodisc/Getty Images

Why do all the rides involve some sort of circular motion? What makes this type of motion so much fun?

Consider taking a ride on the Polar Express. Riders sit side by side on a flat bench seat. As the ride begins to spin faster, the riders slide across the bench seat, to the point where the rider on the left gets squished by the others. The "squishing" force is real and continuous, accelerating the riders constantly yet never changing their speed. Then the ride slows, the force decreases, and the riders are able to slide back to their original seats.

What is the nature of the "squishing" force? How is it related to the velocity of the rider and controlled by the ride operator and the ride designer? The answer to these questions is related to an understanding of inward force and its relationship with radius, mass, and velocity.

As you work through this lesson, keep these questions in mind:

- How do Newton's first and second laws apply to circular motion?
- What factors affect acceleration during circular motion?

Module 5: Lesson 1 Assignments

In this lesson you will complete the Lesson 1 Assignment in the Module 5 Assignment Booklet.

- Try This—TR 1, TR 2, TR 3, and TR 4
- Lab–LAB 1, LAB 2, LAB 3, LAB 4, LAB 5, LAB 6, LAB 7, LAB 8, and LAB 9

The other questions in this lesson are not marked by the teacher; however, you should still answer these questions. The Self-Check and Try This questions are placed in this lesson to help you review important information and build key concepts that may be applied in future lessons.

After a discussion with your teacher, you must decide what to do with the questions that are not part of your

assignment. For example, you may decide to submit to your teacher the responses to Try This questions that are not marked. You should record the answers to all the questions in this lesson and place those answers in your course folder.

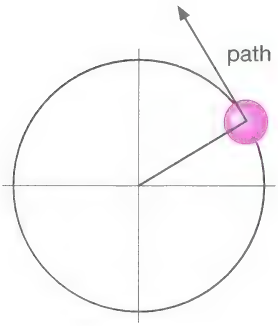
Explore

Visualizing Circular Motion

centripetal: directed toward the centre of a circle

On a circular carnival ride, such as the Polar Express, an inward force acts on the passengers to keep them moving along a circular path. In other words, the seat pushes on the passengers, forcing them to remain on a circular path. An inward force is often called a **centripetal** or centre-seeking force based on its direction.

For simplicity, a simulation will be used in this lesson to investigate and visualize the forces acting on a passenger using a vector diagram of a ball attached to a string moving in a horizontal circle.



Imagine that a ball is being twirled on a string in a **horizontal plane** (as illustrated here). What would happen if the string attached to the ball were cut while it was in motion? Would the ball fly away, stop, or continue on the circular path?

horizontal plane: a plane perpendicular to a radius of Earth, usually used to suggest that there is no vertical component to motion or forces

uniform circular motion: the motion of an object with a constant speed along a circular path

Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "Circular Motion Horizontal" in the search bar. Choose the item called "Circular Motion: Horizontal (Grade 11)." The applet used in this simulation helps you explore the inward force acting on an object travelling with **uniform circular motion**. You can learn more about the simulation and how to use it by reading Show Me found at the top of the simulation screen.

- Select Horizontal Mode (Horizontal Mode). Then use the Initial Velocity "Enter a Slider Value" button ( ) near the bottom of the screen to set the initial velocity at 2.0 m/s.
- Press "Play" ( ), and release the ball using the "Cut" tool ( ).
- You may wish to "Rewind" ( ) the simulation and view the release several times.
- Look at the values for speed, tension, and force shown near the top of the window both before and after the string is cut. Try cutting the string at several different angles.

The direction of the initial velocity is toward the right, but because the direction is continually changing, the speed is what we will be working with.

Self-Check

SC 1. In your own words, explain why the ball moves the way it does once the string has been cut in the simulation.

Check your work with the answer in the appendix.

Newton's First and Second Laws Applied to Circular Motion

Newton's first law of motion says a body continues in its state of rest or of motion in a straight line with a constant speed unless an external, unbalanced force acts on it.

Newton's second law of motion says the rate of change of velocity of an object is proportional to and in the same direction as the unbalanced force acting upon it. Expressed as an equation, it is

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

Both Newton's first and second laws can be applied to circular motion. Newton's first law helps to understand the motion of the ball when the string is cut. The second law helps to understand the motion of the ball when it travels along a circular path.

Module 5: Lesson 1 Assignment

Remember to submit the answer to TR 1 to your teacher as part of your Lesson 1 Assignment in your Module 5 Assignment Booklet.

Try This

TR 1. Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "Circular Motion Horizontal" in the search bar. Choose the item called "Circular Motion: Horizontal (Grade 11)." Use the simulation and Newton's laws to help answer the following questions.

a. When the string is attached to the ball and the ball moves in a horizontal circle, does the string exert a force on the ball? If so, in which direction is the force always directed?

b. While the string is attached to the ball and the ball moves in a horizontal circle, does the force change either or both the direction and speed of the ball? Explain.

c. When the string is released, is there any force acting on the ball?

d. According to Newton's first law, what type of motion should result when the string is released? Is this confirmed by your observations?

e. When a ball is twirling at a uniform speed on the end of a string, the inward force is zero. Is this statement **true** or **false**? Explain your reasoning.

f. An object can be accelerating while maintaining a uniform (constant) speed. Is this statement **true** or **false**? Explain your reasoning.

Read

To help put in context what you learned in the tutorial, read "Defining Circular Motion" on pages 242 to 243 of your textbook.

Self-Check

SC 2. Choose the correct answer. The direction of the velocity vector at any instant in circular motion is always

A. radially in toward the centre
B. a tangent to the circle
C. radially out away from the centre
D. curved around the direction of the circle

Check your work with the answer in the appendix.

Read

Read "Centripetal Acceleration and Force" on pages 243 to 247 of your textbook to see some of the differences between circular motion and the linear motion you have studied in previous units.

Self-Check

SC 3. Design an experiment using the simulation that would investigate the relationship between the speed and the force in uniform circular motion. Describe the manipulated, responding, and controlled variables.

SC 4. What does the word *centripetal* mean from its Latin roots?

Check your work with the answer in the appendix.

Lesson 1 Lab: Inward Force and Circular Motion

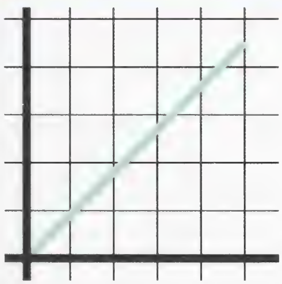
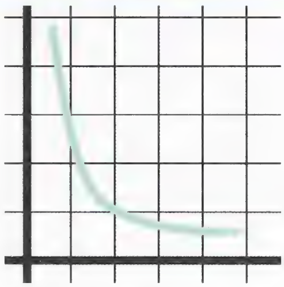
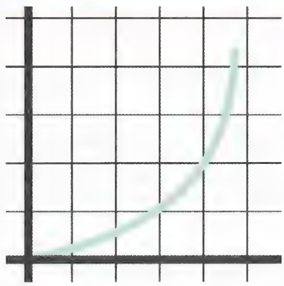
Introduction

When an object travels in a circular path, there must be an inward force causing the direction of the motion to change. This is a direct application of Newton's first and second laws of motion. There is a relationship between the size of the force and the speed of the object, the mass of the object, and the radius of the arc through which the object moves. You can feel the force by whirling an object at the end of a string. This lab activity will be used to investigate and produce a precise mathematical expression that describes these relationships.

What is the precise mathematical relationship between inward force and the variables velocity (v), mass (m), and radius (r) for an object rotating in a horizontal plane?

Prediction

The relationship between the inward force and each variable will show one of three graphical relationships. These reference graphs illustrate the respective mathematical relationships between the variables x and y.

Graphical Analysis Reference Tables		
		
$y \alpha x$ y varies directly as x	$y \alpha 1/x$ y varies as the inverse of x	$y \alpha x^2$ y varies as x squared

Problem 1

How is inward force related to speed?

Procedure

Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms “Circular Motion Horizontal” in the search bar. Choose the item called “Circular Motion: Horizontal (Grade 11).” Re-open the simulation, if necessary, and continue with the procedure.

- Set the mass to 1.0 kg and the radius to 1.0 m.
- Run the simulation, and adjust the initial velocity to the values shown in the table below.

Observations and Analysis

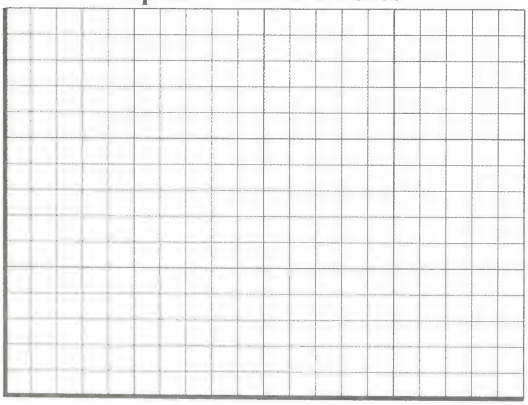
Self-Check

SC 5. Record the inward force (tension) for each speed in the table below.

Speed (m/s)	Inward Force (tension) (N)
1.0	
2.0	
4.0	
8.0	
10.0	
12.0	
15.0	
20.0	
25.0	

SC 6. Use the data from SC 5, and sketch the graph in the space provided below. (**Note:** You may enter this data into a spreadsheet and use the spreadsheet's graphing capabilities.)

Speed vs. Inward Force



SC 7. Compare your graph to the table from SC 5, and state the relationship between the inward force and the speed. Substitute the term F_c for y and the term v for x.

$F_c \propto$ ____

(**Note:** We cannot use the vector sign for the force because the force and speed are not in the same directions. Thus, only the magnitudes are indicated.)

Check your work with the answer in the appendix.

Problem 2

How is inward force related to radius?

Procedure

Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "Circular Motion Horizontal" in the search bar. Choose the item called "Circular Motion: Horizontal (Grade 11)." Use the simulation.

- Set the mass to 1.0 kg.
- Set the initial velocity to 10 m/s.

Module 5: Lesson 1 Assignment

Remember to submit the answers to LAB 1, LAB 2, and LAB 3 to your teacher as part of your Lesson 1 Assignment in your Module 5 Assignment Booklet.

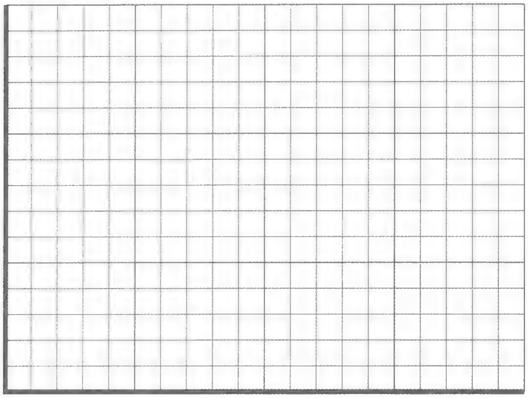
Observations and Analysis

LAB 1. Run the simulation, and adjust the radius to the values indicated in the following table. Record the inward force (tension).

Radius (m)	Inward Force (tension) (N)
0.5	
1.0	
2.0	
3.0	
4.0	
5.0	
6.0	
8.0	
10.0	

LAB 2. Use the data from the table in LAB 1. Sketch the graph in the space provided below. (**Note:** You may enter the data into a spreadsheet and use the spreadsheet's graphing capabilities.)

Radius vs. Inward Force



LAB 3. Compare your graph to the table in LAB 1, and state the relationship between the inward force and the radius. Substitute the term F_c for y and the term r for x.

$F_c \propto$ ______

Problem 3

How is inward force related to mass?

Procedure

Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "Circular Motion Horizontal" in the search bar. Choose the item called "Circular Motion: Horizontal (Grade 11)." Use the simulation.

- Set the radius to 2.0 m.
- Set the initial velocity to 5.0 m/s.

Module 5: Lesson 1 Assignment

Remember to submit the answers to LAB 4, LAB 5, and LAB 6 to your teacher as part of your Lesson 1 Assignment in your Module 5 Assignment Booklet.

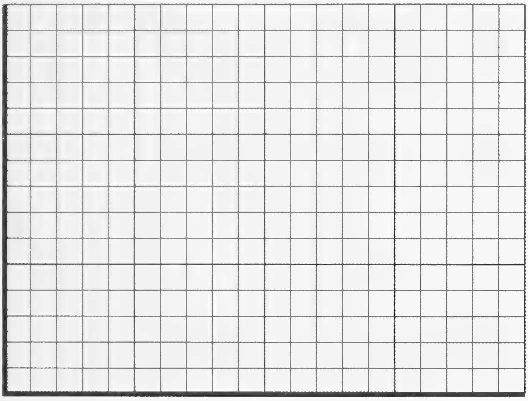
Observations and Analysis

LAB 4. Run the simulation, and adjust the mass to the values indicated in the table below. Record the inward force (tension).

Mass (kg)	Inward Force (tension) (N)
1.0	
2.0	
3.0	
4.0	
5.0	

LAB 5. Use the data from LAB 4. Sketch the graph in the space provided below. (**Note:** You may enter the data into a spreadsheet and use the spreadsheet's graphing capabilities.)

Mass vs. Inward Force



LAB 6. Compare your graph to the table in LAB 4 and state the relationship between the inward force and mass. Substitute the term F_c for y and the term m for x.

$Fc \propto$ ______

Module 5: Lesson 1 Assignment

Remember to submit the answers to LAB 7, LAB 8, and LAB 9 to your teacher as part of your Lesson 1 Assignment in your Module 5 Assignment Booklet.

Conclusion

LAB 7. Combine your answers from SC 7, LAB 3, and LAB 6 to produce a mathematical expression for the inward force as a function of mass, velocity, and radius.

$Fc \propto$ ______

LAB 8. Show that this expression is dimensionally correct according to $F = ma$. In other words, the units for the equation in LAB 8 should be consistent with the fact that 1 N = 1 kg•m/s^2.

LAB 9. Derive an equation for the acceleration acting on an object moving along a circular path in a horizontal plane. (Equate Newton's second law, $F = ma$, to your expression for the inward force from LAB 8.) Go to your Physics 20 Multimedia DVD, and find this equation on the Physics 30 Data Booklet.

Self-Check

SC 8. Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "Circular Motion Horizontal" in the search bar. Choose the item called "Circular Motion: Horizontal (Grade 11)." Use the simulation to confirm the direction of the acceleration vector.

- Set the mass at 1.0 kg and the initial velocity at 11.0 m/s.
- Select the "Vectors" display button (Vectors) near the top of the window. The acceleration vector appears in blue, and the velocity vector appears in pink.
- Play the simulation to see the directions change.

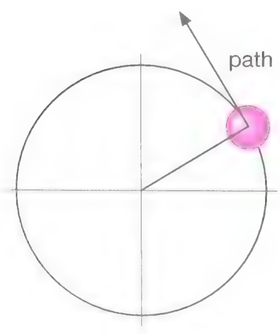
Describe the direction of the following:

a. the acceleration vector
b. the velocity vector

Check your work with the answer in the appendix.

Read

Read pages 248 to 250 of the textbook, starting at "Circular Motion and Newton's Laws." Look for the new terms to describe the speed of rotation.



The speed of rotation is usually measured by the time it takes for one revolution or cycle, called the **period** (T), or by the **frequency** (f) of revolutions or cycles in a time period. Frequency may be expressed as rpm (revolutions per minute) or in the SI units of hertz (Hz).

1 Hz = 1 cycle/s

$$T = \frac{1}{f} \text{ and } f = \frac{1}{T}$$

period: the time for one complete cycle (T)

frequency: the number of cycles in a time period (f)

Self-Check

SC 9. Complete question 3 of the "Practice Problems" on page 250 of the textbook.

Check your work with the answer in the appendix.

Read

Normally we think of speed in terms of km/h or m/s. How do frequency in Hz and period (*T*) relate to that? Read pages 250 and 251 of the textbook, starting at "Speed and Circular Motion," to learn how to convert from one to the other.

Self-Check

SC 10. Complete question 1 of the "Practice Problems" on page 251 of the textbook.

Check your work with the answer in the appendix.

Read

What about acceleration and force? How do they apply to circular motion? Read pages 252 to 255 of the textbook to see how acceleration is handled in these situations.

Self-Check

SC 11. Complete question 1 of the "Practice Problems" on page 255 of your textbook.

Check your work with the answer in the appendix.

Read

In the simulation Circular Motion: Horizontal, which used the ball and string earlier in the lesson, the force was just given in the data. How is force calculated for circular motion? Read page 256 of the textbook to see how force is determined in circular motion.

Self-Check

SC 12. Complete question 2 of the "Practice Problems" on page 256 of your textbook.

Check your work with the answer in the appendix.

Newton's laws can be applied to many different situations where circular motion is common, such as travelling in a vehicle. In such cases, there are many different forces that can be directed inward. For a car rounding a curve, it is the friction between the road and the tires that is directed inward. On a carnival ride, the inward force is tension in the structure (similar to that of a ball and string). Go to your Physics 20 Multimedia DVD, and complete the tutorial called "Horizontal Circular Motion: Forces." Completing this tutorial will help you explore the nature and types of forces that cause circular motion.

Self-Check

SC 13. Suppose you were to triple the speed with which you twirled a ball on a string. How would that affect the size of the inward force (tension) in the string?

Check your work with the answer in the appendix.

Module 5: Lesson 1 Assignment

Remember to submit the answers to TR 2 and TR 3 to your teacher as part of your Lesson 1 Assignment in your Module 5 Assignment Booklet.

Try This

TR 2. Suppose you were to reduce the radius of the arc through which you twirled a ball on a string by one-half. How would that affect the size of the inward force (tension) in the string?

TR 3. Suppose you were to increase the mass of the ball by five times. How would that affect the size of the inward force (tension) in the string?

Self-Check

SC 14. Complete question 2 of "5.1 Check and Reflect" on page 247 of the textbook.

Check your work with the answer in the appendix.

Module 5: Lesson 1 Assignment

Remember to submit the answer to TR 4 to your teacher as part of your Lesson 1 Assignment in your Module 5 Assignment Booklet.

Try This

TR 4. Complete question 8 of "5.2 Check and Reflect" on page 268 of the textbook.

Discuss

Imagine you have a ball on the end of a rope and you whirl it in a horizontal circle at a steady speed. Does it feel like the ball is pulling on your hand, or does it feel like your hand is pulling on the ball? Or do two forces exist as an action-reaction pair in the case of Newton's third law? If two forces exist, which force is the action and which is the reaction?

Confusion about these questions leads to a common misconception that circular motion is caused by a centrifugal force. The term *centrifugal* is based on the Latin terms *centrum* ("centre") and *fugere* ("to flee"), making this an outward force. In contrast, the term *centripetal* is based on the Latin terms *centrum* ("centre") and *petere* ("tend towards"), making this an inward force.

In the discussion forum, propose an answer to each of these two questions:

D 1. Which term, *centripetal* or *centrifugal* correctly describes the action force and, therefore, the real force that causes circular motion?

D 2. Explain why the term *centrifugal* is sometimes used to describe the cause of circular motion but does not represent a force that actually exists.

Reflect and Connect


© Steven Pepple/shutterstock

When engineers designed the ride pictured here, they set up the passenger seating so that everyone would be facing inward. This was intentional. The inward (centripetal) force will be significant; so, pushing from behind is the safest way to apply this force in these circumstances. The tension in the arms of the structure produces the inward force, which acts on the passengers and forces them to travel in a circle. The ride will also have a maximum safe speed, which is based on the amount of tension that can be applied in the structure and handled by the riders. This is also considered when determining the length of each arm that holds the seats. When loading passengers, the operators have to be careful to balance the mass in the system. As well, keeping an equal number of riders on each swing is important to maintain an equal tension in each of the arms of the spinning ride. At higher speeds, things gets exciting, with the inward force accelerating the passengers from behind and constantly changing the direction of their velocity as they see the world below spinning quickly. It is the acceleration that puts the thrill in the ride and makes the engineering specifications so detailed.

Compare this design to that of the Polar Express ride, where passengers face the direction of travel rather than the centre of the circular path. In such a case, the maximum inward force will be much smaller so that the outside rider is squished, but not crushed, by those on the inside.

Answer the following question, and store your answer in your Physics 20 course folder: How do the ride engineers and operators ensure the inward force is smaller on rides such as the Polar Express?

Reflect on the Big Picture

Each of the Reflect on the Big Picture sections in this module will ask you to consider the movement of objects in a circle or part of a circle. To help you reflect on your learning from this lesson, complete at least one of these reflection activities:

- You've read about carnival rides that spin and push people to the outside. You've likely experienced the same forces just riding around in cars or buses. Create a poem or short story about being pushed around by these invisible forces. Present your creation in either an oral or written form.
- You've probably heard someone say that watching a spinning skater makes them dizzy. Research how skaters doing rapid spins overcome such dizziness, and write a short report on it. You may choose to share your research with others in the discussion area and use any comments you get from your classmates to assist you in writing your report.

Store your completed reflection in your Physics 20 course folder.

Module 5: Lesson 1 Assignment

Make sure you have completed all of the questions for the Lesson 1 Assignment. Check with your teacher about whether you should submit your assignment now or wait until all of the Module 5 assignments have been completed.

Lesson Summary

This lesson focused on the solution of these questions:

- How do Newton's first and second laws apply to circular motion?
- What factors affect acceleration during circular motion?

When an object travels in a circular path, there must be an inward force causing the direction of the motion to change. The inward force can be friction, tension, or gravity, for example. This is a direct application of Newton's first and second laws of motion. It may also be observed, by whirling an object at the end of a string, that there is a relationship between the size of the force and the speed of the object, the mass of the object, and the radius of the arc through which the object moves. Such relationships can be applied to the design and operation of objects that move in horizontal circular paths, such as those commonly experienced in carnival rides.

Inward (centripetal) force can be mathematically described by the relationship among velocity, mass, and radius for an object rotating in a horizontal plane. Expressed as an equation,

$$F_c = \frac{mv^2}{r}$$

Quantity	Symbol	SI Unit
inward (centripetal) force	F_c	N
mass	m	kg
radius of circular path	r	m
speed	v	m/s

Applying Newton's second law to the equation for centripetal force gives the following expression for centripetal acceleration:

$$ma = \frac{mv^2}{r}$$

$$a_c = \frac{v^2}{r}$$

Lesson Glossary

centripetal: directed toward the centre of a circle

frequency: the number of cycles in a time period (f)

horizontal plane: a plane perpendicular to a radius of Earth, usually used to suggest that there is no vertical component to motion or forces

period: the time for one complete cycle (T)

uniform circular motion: the motion of an object with a constant speed along a circular path

Lesson 2—Newton's Laws Applied to Vertical Circular Motion

© Jim Lopes/shutterstock

Get Focused

Roller coasters can be extremely exciting. At the very top of this loop, at just the right speed, the riders travelling in the roller coaster will feel **weightless**. A moment later, as the coaster comes to the bottom of the loop, the riders will feel much heavier than usual. This is achieved by dramatic shifts in the net acceleration of the riders, giving them the sensation of changing weight. You may experience the same sensation while riding on a Ferris wheel, although to a lesser extreme. In both cases, understanding the motion and the sensations is based on similar principles.

An application of free-body analysis and Newton's second law can be used to design vertical loops that maintain a **tension** that produces just the right amount of acceleration for a good adrenaline rush. What is the right amount of acceleration? What is the difference between the acceleration of the rider at the top of a vertical loop and his or her acceleration at the bottom? How do you design the loop to produce just the right amount of acceleration for the riders?

As you work through this lesson, keep these important questions in mind:

- How do Newton's laws explain the sensations during vertical circular motion, such as in a loop on a roller coaster?
- What forces come into play during vertical circular motion?

weightless: experiencing little apparent gravitational pull

tension: a stress that tends to stretch an object

Module 5: Lesson 2 Assignments

In this lesson you will complete the Lesson 2 Assignment in the Module 5 Assignment Booklet.

- Try This—TR 1, TR 2, TR 3, TR 4, and TR 5

You must decide what to do with the questions that are not marked by the teacher. Remember that these questions provide you with the practice and feedback that you need to successfully complete this course. You should respond to all the questions and place those answers in your course folder.

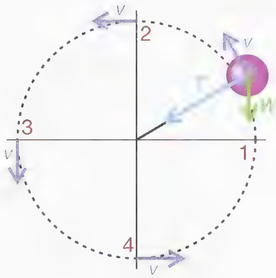
Explore

> **vertical circular motion:** motion in a circular path where one diameter of the circle is vertical

Using vector diagrams and Newton's second law, **vertical circular motion** can be understood in terms of the acceleration acting on an object or rider that moves along a vertically circular path. For simplicity, a simulation will be used in this lesson to produce free-body diagrams for an object attached to a massless, rigid rod travelling in a vertical, circular path. Then, using the free-body diagrams, you will derive equations describing the inward force and tension at the top of the circle, at the bottom of the circle, and at any other position along the circle. From this analysis, you will be able to determine the acceleration acting on an object or rider at any point in the vertical circle.

Visualizing Circular Motion: Vector Diagrams

Imagine that a ball is being twirled on a rigid, massless rod in a vertical plane (as shown in the vertical plane on the right). A simulation will be used to illustrate this situation.



You will be using an applet that simulates the motion of a mass on a rod that is moving along a vertical circular path. You can use its free-body analysis to explore the equations describing the inward force and tension at any point along the arc.

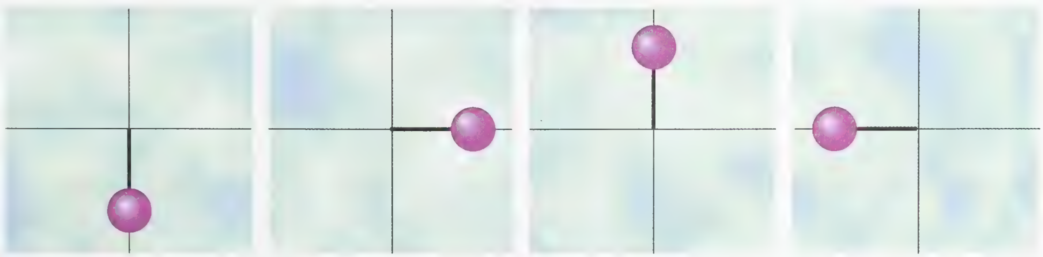
Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "Circular Motion Vertical" in the search bar. Choose the item called "Circular Motion: Vertical (Grade 11)."

Start the simulation. On the simulation, select "Vertical Mode" (Vertical Mode) and "Vectors" (Vectors). Next, use the "Enter a slider value" button ( ) below the Initial Velocity slider near the bottom centre of the screen to set the initial velocity to 9.0 m/s. Press "Play." Observe the motion of the ball, and watch the labels of vectors on the diagram:

- velocity(v), shown in magenta and always perpendicular to the rod
- weight (W), shown in green and always directed downward, is the gravitational force
- tension (T), shown in blue and always directed inward

Self-Check

SC 1. Use the simulation. While the ball is revolving, observe the magnitude (size) of the velocity vector and the corresponding speed measurement at the top of the simulation. Label each of the following diagrams with the velocity vector. (Remember to illustrate the relative magnitude of each vector.)



SC 2. Based on your drawing, explain how the velocity is changing in terms of

a. direction
b. magnitude

Check your work with the answer in the appendix.

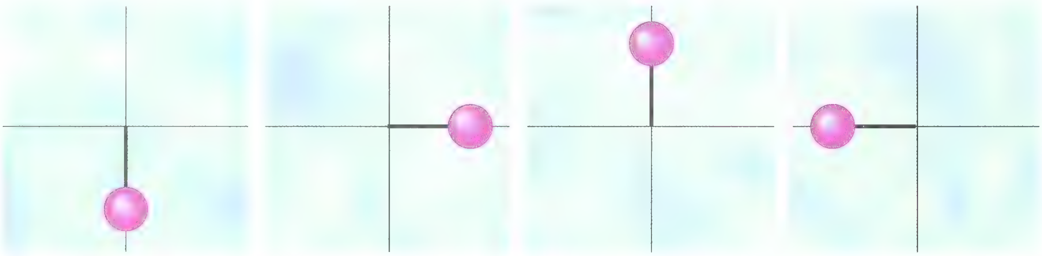
Module 5: Lesson 2 Assignment

Remember to submit the answer to TR 1 to your teacher as part of your Lesson 2 Assignment in your Module 5 Assignment Booklet.

Try This

TR 1. Use the simulation, and observe the tension vector while the ball is rotating.

a. Label each of the following diagrams with the tension vector. (Remember to illustrate the relative magnitude of each vector.)



b. Identify which diagram has the greatest tension force, which diagram has the least tension force, and which two diagrams have the same magnitude of tension force.

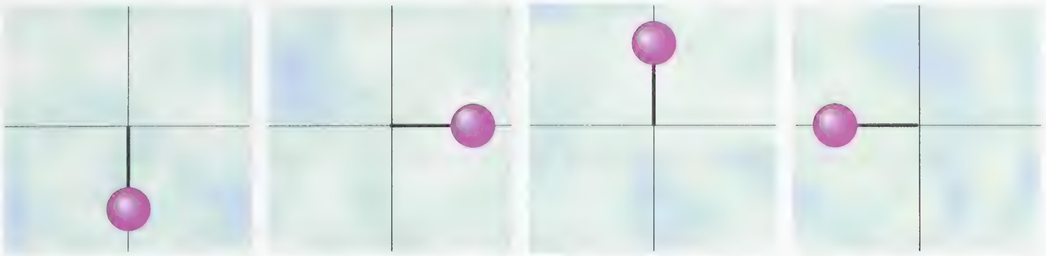
c. The tension force is always directed towards the ______ of the circle.

d. The tension force is greatest at the ______ of the circle and least at the ______ of the circle.

Self-Check

SC 3. Use the simulation, and observe the weight vector while the ball is rotating.

a. Label each of the following diagrams with the weight vector. (Remember to illustrate the relative magnitude of each vector.)



b. The weight (gravitational force) is always directed ______.

c. The magnitude and direction of the weight vector does not _______.

Check your work with the answer in the appendix.

Go to your Physics 20 Multimedia DVD, and complete the tutorial called "Vertical Circular Motion: Forces." Explore the nature and types of forces involved in circular motion by completing this tutorial.

Understanding the Forces That Produce Circular Motion in a Vertical Plane

Previously, a simulation was used to investigate the direction and magnitude of the velocity, tension, and weight (gravitational force). Next, a free-body diagram (FBD) analysis will be used to determine the mathematical relationships among velocity, tension, and weight.

Three special cases will be illustrated:

- when the ball is at the top of the arc
- when the ball is at the bottom of the arc
- when the ball is at any position along its path (Going Beyond)

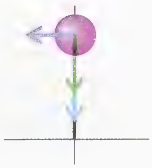
In all cases, there are only two forces:

- the tension force in the rod
- the gravitational force or weight of the object

Ignore the magenta-coloured velocity vector. It is not part of a true FBD and is only there to indicate speed and direction of motion.

Case 1: At the Top of the Arc

Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms “Circular Motion Vertical” in the search bar. Choose the item called “Circular Motion: Vertical (Grade 11).” Restart the simulation. Press “Pause” when the ball reaches the top of the circle (as shown).



Turn on the vector free-body diagram display using the “Vectors” button (). At the infinitesimal instant that the ball is at the top of the circle, all of the forces are acting vertically.

Self-Check

SC 4. Define the positive direction as upward, and carefully label the tension force and weight force on the figure. What direction does each of these forces have in this case? Use + or – to label the direction on the diagram.

Check your work with the answer in the appendix.

Module 5: Lesson 2 Assignment

Remember to submit the answer to TR 2 to your teacher as part of your Lesson 2 Assignment in your Module 5 Assignment Booklet.

Try This

TR 2. The inward force is the force needed to deflect the ball in a circular path. At the top of the arc, F_{inward} is the net force or sum of the weight and tension forces. Use the vector diagram of the ball at the top of the circle to answer the following questions.

a. Using the FBD for the ball at the top of the circle, write an equation for F_{inward}.

$F_{\text{inward}} =$ ________ + ________

b. What is the direction of the inward force at the top of the arc?

c. Rewrite the equation of F_{inward}, and include the direction of all of the forces. (Notice that at this infinitesimal instant, all of the forces are along the y-axis; so, you can use + or – to represent direction.)

________ = ________ + ________

d. Manipulate this expression in terms of the tension force (T) in the rod.

$T =$ ________ + ________

e. An inward force can also be defined by the expression $F_{\text{inward}} = \frac{mv^2}{r}$. Remember that weight is defined as $W = mg$. Rewrite the equation from TR 2.d. by substituting these expressions.

Go to your Physics 20 Multimedia DVD, and complete the tutorial called "Vertical Circular Motion: Forces at the Top." Explore how speed varies during vertical circular motion by completing this tutorial.

In screen 4, after you have made the calculations, click on the rectangles at the bottom of the choices that you think are correct.

Self-Check

SC 5. Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "Circular Motion Vertical" in the search bar. Choose the item called "Circular Motion: Vertical (Grade 11)." Restart the simulation. Click the "Vectors" (Vectors) box and the "Vertical mode" (◉ Vertical Mode) button at the top of the screen. Set up the following conditions in the simulation. The mass and radius settings can be adjusted by moving the sliders. Double-clicking the slider or clicking ( ) allows you to enter an exact value for the variable.

- mass of the ball = 2.0 kg
- initial velocity = 12.0 m/s
- radius of the arc = 2.0 m

Press "Play." Then press "Pause" when the ball is near the top of the arc. Click the ball, and position it at the exact top of the arc so the angle shows 90.0°.

a. Record the speed at the top of the arc: _______ m/s

b. Calculate the expected tension in the rod at the top of the arc by doing the following:

   i. State the equation you derived in TR 2.e.
   ii. Calculate the tension acting at the top of the vertical loop using the mass, radius, and velocity at the top of the arc.
   iii. Verify your answer by checking the tension measurement on the simulation when the ball is positioned at the top of the arc.

   tension measurement: ________ N

Check your work with the answer in the appendix.

Module 5: Lesson 2 Assignment

Remember to submit the answers to TR 3 and TR 4 to your teacher as part of your Lesson 2 Assignment in your Module 5 Assignment Booklet.

Try This

TR 3. Set up the following conditions in the simulation. (The mass and radius settings can be adjusted by moving the sliders. Double-clicking the slider or clicking () allows you to enter an exact value for the variable.)

- mass of the ball = 3.0 kg
- initial velocity = 18.0 m/s
- radius of the arc = 1.0 m

Press "Play." Then press "Pause" when the ball is near the top of the arc. Click the ball, and position it at the exact top of the arc so the angle shows 90.0°.

a. Record the speed at the top of the arc: _______ m/s

b. Calculate the expected tension in the rod at the top of the arc by doing the following:

   i. State the equation you derived in TR 2.e.
   ii. Calculate the tension acting at the top of the vertical loop using the mass, radius, and velocity at the top of the arc.
   iii. Verify your answer by checking the tension measurement on the simulation when the ball is positioned at the top of the arc.

   tension measurement: _______

TR 4. Set up the following conditions in the simulation. (The mass and radius settings can be adjusted by moving the sliders. Double-clicking the slider or clicking () allows you to enter an exact value for the variable.)

- mass of the ball = 1.0 kg
- initial velocity = 7.0 m/s
- radius of the arc = 1.0 m

Press "Play." Then press "Pause" when the ball is near the top of the arc. Click the ball and position it at the exact top of the arc so the angle shows 90.0°.

a. Record the speed at the top of the arc: _______ m/s.

b. Calculate the expected tension in the rod at the top of the arc by doing the following:

   i. State the equation you derived in TR 2.e.
   ii. Calculate the tension acting at the top of the vertical loop using the mass, radius, and velocity at the top of the arc.
   iii. If the tension in the rod reaches zero at the top of the arc, the object is in a momentary state of free fall. Assuming the tension is zero, use the equation from TR 2.e. to derive an expression for the velocity at the top of the arc when an object undergoes free fall.
   iv. Explain what this equation will be used to calculate. Refer to a bucket of water in your explanation.

   tension measurement: _______

Self-Check

SC 6. A 2.50-kg ball is attached to a 3.00-m bar and swung in a vertical circle.

a. If the ball does not leave the circular loop, what minimum speed must it have at the top of the arc?

b. By adjusting the initial velocity slider on the simulation when the radius is set to 3.0 m, find the initial velocity required to make the ball undergo free fall at the very top of the vertical loop.

   initial velocity: _________ m/s

c. During your exploration in SC 6.b., you may have noticed that the bar can turn red when the ball moves very slowly at the top of the loop. What does this colour change symbolize? (**Hint:** Look at the tension measurement when the bar changes colour.)

Check your work with the answer in the appendix.

Read

How do you explain the weightless feeling you get on a roller coaster during an upside-down loop? Read "A Vertical System in Circular Motion" on pages 260 to 262 of the physics textbook to get some answers.

Self-Check

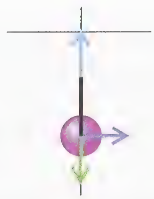
SC 7. Complete question 2 of "Practice Problems" on page 262 of the textbook.

Check your work with the answer in the appendix.

Case 2: At the Bottom of the Arc

Self-Check

SC 8. This figure shows the ball at the bottom of the arc. At the infinitesimal instant that the ball is at the bottom of the circle, all of the forces are acting vertically. Define the positive direction as upward, and carefully label the tension force and weight force on this diagram. What direction does each of these forces have in this case? Use + or − to label the direction on the diagram.



SC 9. The inward force is the force needed to deflect the ball in a circular path. At the bottom of the arc, F_{inward} is the net force or sum of the weight and tension forces.

a. Using the FBD constructed in SC 8, write an equation for F_{inward}.

$F_{\text{inward}} =$ ________ + ________

b. What is the direction of the inward force at the bottom of the arc?

c. Rewrite the F_{inward} equation, and include the direction for all of the forces. (Notice that at this infinitesimal instant, all of the forces are along the y-axis; so, you can use + or – to represent direction.)

________ = (________) + (________)

d. Manipulate this expression in terms of the tension force (T) in the rod.

$T =$ ________ + ________

e. An inward force can also be defined by the expression $F_{\text{inward}} = \dfrac{mv^2}{r}$. Remember that weight is defined as $W = mg$. Rewrite the equation from SC 9.d. by substituting these expressions into it.

Check your work with the answer in the appendix.

Go to your Physics 20 Multimedia DVD, and complete the tutorial called "Vertical Circular Motion: Forces at the Bottom." Explore the tension at the bottom of a vertical circular motion in this tutorial.

In order to complete this tutorial, you must have Authorware Player, which you will have downloaded for the previous two tutorials.

The tutorial uses the term F_{inward} for the *centripetal force vector*, $\vec{F}_c$, and omits the vector signs above the symbols. It states that they are vectors and adds them like you should for a vector.

In screen 8, if you can't see the second-last quantity asked for, it is the acceleration due to gravity, g.

Read

To get a summary of what you have learned about forces in circular motion so far, read from the bottom of page 262 through page 264 of the physics textbook.

Comparing the Different Forms of the Tension Equation

The textbook uses the following formula for the *magnitude* of the tension at the bottom of the circle:

$T = \dfrac{mv^2}{r} - (-mg)$ (It substitutes (+9.81 m/s^2) for g.)

The tutorial uses the following *vector* equation, even though the vector signs are not used:

$$T = \frac{mv^2}{r} - mg$$

The tutorial substitutes in $-9.81\ \text{m/s}^2$ for g. Can you see that gives an equivalent result to the text because subtracting a negative number is the same as adding a positive number?

In SC 9.e. you arrived at the following equation for the *magnitude* of the tension at the bottom of the circle:

$$T = \frac{mv^2}{r} + mg$$

There you use $+9.81\ \text{m/s}^2$ for g because you are again dealing with magnitudes. Can you see that it will give an equivalent result to the other two equations?

The important thing to know is that the magnitude of the tension at the bottom of the circle is larger than the magnitude of centripetal force because the force of gravity must be added. At the top of the circle, the magnitude of the tension is smaller than the magnitude of the centripetal force because the force of gravity must be subtracted. At the horizontal positions on either side of the circle, the tension is equal to the centripetal force because the force of gravity is exactly perpendicular to the tension and the centripetal force.

Self-Check

SC 10. Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "Circular Motion Vertical" in the search bar. Choose the item called "Circular Motion: Vertical (Grade 11)." Restart the simulation. Set up the following conditions in the simulation. (The mass and radius settings can be adjusted by moving the sliders. Double-clicking the slider or clicking ( ) allows you to enter an exact value for the variable.)

- mass of the ball = 2.0 kg
- initial velocity = 12.0 m/s
- radius of the arc = 2.0 m

Press "Play." Then press "Pause" when the ball is near the bottom of the arc. Click the ball, and position it at the exact bottom of the arc so the angle shows 270.0°.

a. Record the speed at the bottom of the arc: ______ m/s

b. Calculate the expected tension in the rod at the bottom of the arc by doing the following:

   i. State the equation you derived in SC 9.e.
   ii. Calculate the tension acting at the bottom of the vertical loop using the mass, radius, and velocity at the bottom of the arc.
   iii. Verify your answer by checking the tension measurement on the simulation when the ball is positioned at the bottom of the arc.

   tension measurement: _______ N

Check your work with the answer in the appendix.

Module 5: Lesson 2 Assignment

Remember to submit the answer to TR 5 to your teacher as part of your Lesson 2 Assignment in your Module 5 Assignment Booklet.

Try This

TR 5. Complete question 3 in "Practice Problems" on page 264 of the textbook.

Read

What happens when the rate of rotation is given as frequency or period instead of speed? How are the circular motion equations adapted to accommodate this? Read "Centripetal Force, Acceleration, and Frequency" on pages 265 to 267 of the physics textbook.

Self-Check

SC 11. Complete question 1 of "Practice Problems" on page 267 of the physics textbook.

Check your work with the answer in the appendix.

Reflect and Connect

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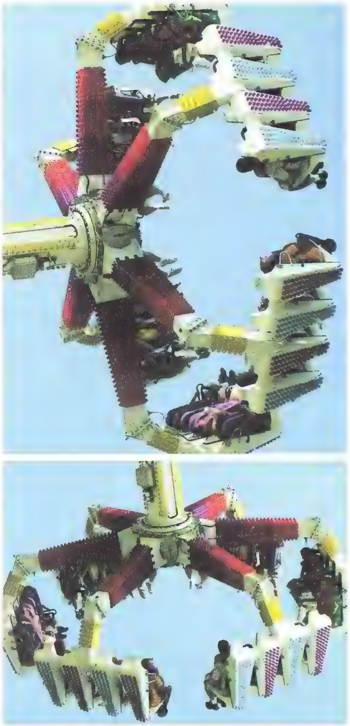
What makes vertically circular rides so much fun? On a Ferris wheel, for example, the riders at the top experience an inward force that is due to gravity while the tension in the steel opposes it (but not completely). This gives you the sensation that your apparent weight is dropping, since the tension acting on you is less than the force of gravity, which makes you feel light as you travel downward along the arc. On a roller coaster, at just the right speed, the tension in the steel is zero at the top, allowing the riders to free fall and experience weightlessness as they descend. At greater speeds, the tension can act inward, pulling the rider into the seat and accelerating them downward at a rate greater than that of gravity alone. What you experience on the ride is directly related to the amount of tension acting on you as the ride moves.

At the bottom of a roller coaster or Ferris wheel, the inward force is directed upwards and caused by the tension in the steel. At this point, the force of gravity opposes the inward tension. Since it is the tension at the bottom that you feel, it has to be greater than that of gravity in order to keep the riders on a circular path. As a result, the riders experience acceleration larger than gravity, making them feel heavy. At higher speeds, the tension is greater and so, too, is the heavy sensation. At higher speeds, in general, all the forces are larger, which makes the use of seat belts a common safety feature on high-speed rides, such as roller coasters, but not on low-speed rides, such as Ferris wheels.

Discuss

Some thrill rides offer both horizontal and vertical motion at different points in the ride. In the discussion forum, explain the change in sensation that the people in the rides pictured here experience as they move from a horizontal position to a vertical position.



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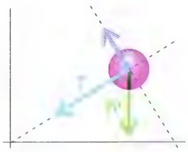
Reflect on the Big Picture

Each of the Reflect on the Big Picture sections in this module will ask you to consider the movement of objects in a circle or part of a circle. Complete at least one of these reflection activities:

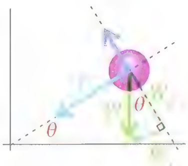
- Create a drawing, painting, or multimedia presentation that describes the sensations of roaring through a loop on a roller coaster.
- Research the effects of apparent increases and decreases of gravity on the human body. Look for ways to prepare for the sensations you experience on carnival rides.

Store your completed reflection in your Physics 20 course folder.

Going Beyond



How is vertical circular motion explained at positions that are not at the top or bottom of the circular path? Consider the free-body diagram here. A coordinate axis (dashed lines) is introduced to define the directions. Notice that the weight is not on the same axis as the tension; therefore, the magnitude of the inward force cannot be found by simply adding the magnitude of the weight and tension.



To determine the inward force, you must first resolve the weight into a component that acts in the same direction as the tension force and a component that acts at right angles to it. The component of weight, labelled W_1, acts parallel to the tension (towards the centre of the circle). Therefore, the inward force can be found by adding the tension and parallel weight component. For example,

$$F_{\text{inward}} = T + W_1$$
$$F_{\text{inward}} = T + W \sin\theta$$

Can you show that the expression for the tension, in this case, is given by the equation $T = \frac{mv^2}{r} - mg\sin\theta$?

Do the special cases of $\theta = 270°$ and $\theta = 90°$ give expressions that are equivalent to the equations derived for the top and the bottom of the vertical circle?

If the ball rotates 270° counterclockwise from the positive x-axis, the ball will be at the bottom of the circle and sin (270°) = −1. Therefore, the equation becomes

$$T = \left(\frac{mv^2}{r}\right) + (mg)$$

If the ball rotates 90° counterclockwise from the positive x-axis, the ball will be at the top of the circle and sin (90°) = +1. Therefore, the equation becomes

$$T = \frac{mv^2}{r} + (-mg)$$

Module 5: Lesson 2 Assignment

Make sure you have completed all of the questions for the Lesson 2 Assignment. Check with your teacher about whether you should submit your assignment now or wait until all of the Module 5 assignments have been completed.

Lesson Summary

In this lesson you focused on the solution to these questions:

- How do Newton's laws explain the sensations during vertical circular motion, such as during a loop in a roller coaster?
- What forces come into play during vertical circular motion?

Free-body analysis can be used to understand the net inward force and tension acting on an object at the top and bottom of a vertical circular path. The tension, in turn, can be related to the sensations that a rider will experience while moving on a vertical circular path, such as those experienced on many roller coasters. At the top of the vertical circle, the tension acting is described by

$T = \frac{mv^2}{r} + (-mg)$, where $g = +9.81\ \text{m/s}^2$

If the tension reaches zero at the top of the arc, the object is in a momentary state of free fall. The speed at which this will occur is described by

$$0 = \frac{mv^2}{r} + (-mg)$$

$$\frac{\cancel{m}v^2}{r} = \cancel{m}g$$

$$v = \sqrt{gr}$$

At the bottom of the vertical circle, the tension acting is greater because it opposes the force due to gravity.

$$T = \left(\frac{m v^2}{r}\right) + (m g)\text{, where } g = +9.81\ \text{m/s}^2$$

From this analysis, you are able to determine the acceleration acting on an object or rider at any point in the vertical circle, leading to a better understanding of the causes of the sensations experienced by riders when they are moving in vertical circles.

Lesson Glossary

tension: a stress that tends to stretch an object

vertical circular motion: motion in a circular path where one diameter of the circle is vertical

weightless: experiencing little apparent gravitational pull

Lesson 3—Kepler's Three Laws of Planetary Motion

Get Focused

© Andrea Danti/shutterstock

Astronomy is the science of celestial objects, such as the planets, stars, galaxies, and other objects outside Earth's atmosphere. In particular, astronomy concerns the evolution and motion of celestial objects that provides valuable information about the developing universe. Today's astronomical data is vast, with high-quality data being gathered by instruments like the *Hubble Space Telescope*. This is a change from the hilltop observations made with the unaided eye prior to Galileo's innovative use of the telescope. The seventeenth century saw astronomy take a giant leap forward when Johannes Kepler's three laws successfully described the motion of the planets. Soon after Kepler's descriptions of planetary motion had been completed, Sir Isaac Newton's explanation of planetary motion followed. Astronomy had officially become a crowning achievement of western science. Exactly how did people come to understand the motion of the solar system in the 1600s?

As you work through this lesson, keep this question in mind:

- How did Kepler's laws help calculate the position of the planets as they orbited the Sun?

Module 5: Lesson 3 Assignments

In this lesson you will complete the Lesson 3 Assignment in the Module 5 Assignment Booklet.

- Try This—TR 1, TR 2, TR 3, TR 4, TR 5, TR 6, TR 7, TR 8, and TR 9

You must decide what to do with the questions that are not marked by the teacher.

Remember that these questions provide you with the practice and feedback that you need to successfully complete this course. You should respond to all the questions and place those answers in your course folder.

Watch and Listen

Go to your Physics 20 Multimedia DVD, and watch the video called "Johannes Kepler." This video will explain both his life and his life's work.

Explore

Kepler's Three Laws of Planetary Motion

Despite the difficulties in his personal life, Kepler (1571–1630) formulated the three laws of planetary motion. These laws describe the manner in which the planets move around the Sun, but they do not explain why the planets move in this fashion. This explanation was provided later by Isaac Newton (1642–1727).

elliptical: having the shape of an ellipse or oval

epicycle: a circle that rolls along the circumference of another circle

Kepler's three laws are based on observations by Tycho Brahe (1546–1601). They allowed a great simplification and increased accuracy in the calculation of the planets' motions compared to the earlier Copernican model. This advance was brought about when Kepler realized that planets were moving in **elliptical** orbits rather than on **epicycles** of circles as in the Copernican model.

Kepler's laws are now known to be only an approximation, but a very good one. The observed deviations of the planets' motions from Kepler's laws can mostly be explained in terms of Newton's laws of motion applied to the planets in combination with his universal law of gravitation.

The Newtonian analysis shows that Kepler's laws would be true if the gravitational force exerted on the planets by the Sun was the only force acting on the planets. However, the planets also exert gravitational forces on each other; and it is these additional small forces that cause the planets' motions to be slightly different from those predicted by Kepler's laws.

Newton's theory can explain these deviations from Kepler's laws except for one remaining small discrepancy, which was accounted for early in the twentieth century by Albert Einstein's theory of general relativity.

Watch and Listen

Go to your Physics 20 Multimedia DVD, and watch the video called "Kepler and Tycho Brahe."

Data in Astronomical Units

The study of planetary motion, by necessity, involves distances in space that are incredibly large. For this reason, distance measurements in space are not always made using conventional distance units, such as metres. The AU is the **astronomical unit**. It is a unit of distance that is equal in length to the semi-major axis (shown in red in the ellipse at the right) of Earth's orbit around the Sun (the mean or average distance between Earth and Sun).

$$1\ \text{AU} = 1.496 \times 10^{11}\ \text{m}$$

astronomical unit: the average distance between Earth and the Sun

A simulation that uses astronomical units will be used to explore Kepler's three laws and show how they accurately describe planetary motion.

The applet used in this simulation allows you to

- visualize the relative motion of a planet orbiting the Sun
- vary orbital eccentricity
- explore Kepler's laws

The year used in the simulation is the **sidereal year**, which is the time it takes Earth to make one revolution around the Sun relative to the fixed stars.

1 Year = 365.26 mean solar days

sidereal year: the orbital period of Earth

calendar year: 365 days or 366 days (leap year)

The sidereal year must be distinguished from the **calendar year**, which is equal to 365 or 366 mean solar days, depending on the particular year. The abbreviation for the sidereal year is either the letter y or the letter a (as in annual).

Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "Planetary Motion" in the search bar. Choose the item called "Planetary Motion (Grade 11)." Open the silmulation. You can learn more about the simulation and how to use it by reading Show Me found at the top of the simulation screen. To view the astronomical data on the simulation, click the "Data" button ( ). Observe that various quantities are displayed in units of AU and Year. On the simulation, some of the quantities are set by sliders that also use the astronomical and year units. They are convenient units in an astronomical context.

energy/mass (E/m):	-19.74 AU^2/Year^2
angualr momentum/mass (L/m):	5.51 AU^2/Year
areal velocity (dA/dt):	2.75 AU^2/Year
angular velocity (ω):	4.52 rad/Year
time (t):	0.21 Year
distance (r):	1.10 AU
speed (v):	5.68 AU/Year
acceleration (a):	32.40 AU/Year^2

Self-Check

SC 1. If time (t) is given in years and distance (r) is given in astronomical units (AU), substitute these units into the formula below to determine the units of speed for this simulation.

$$\text{speed } (v) = \frac{\text{distance } (r)}{\text{time } (t)}$$
$$= \underline{\hspace{3cm}}$$

The units for change in velocity are the same as for speed. Use this fact to determine the units of acceleration in the formula below.

$$\text{acceleration } (\vec{a}) = \frac{\text{change in velocity } (\Delta \vec{v})}{\text{time } (t)}$$
$$= \underline{\hspace{6em}}$$

Verify that your units for speed and acceleration are consistent with those displayed in the table above or click the "Data" button in the simulation.

Check your work with the answer in the appendix.

Kepler's First Law

Module 5: Lesson 3 Assignment

Remember to submit the answer to TR 1 to your teacher as part of your Lesson 3 Assignment in your Module 5 Assignment Booklet.

Try This

TR 1. Use the simulation to investigate Kepler's first law.

- Click the "Reset" button ( ).
- Using the eccentricity slider, set the eccentricity (e) to 0.400.
- Using the semi-major axis slider (mean distance between the planet and the Sun), set a to 2.0 AU.
- Click the "Play" button ( ), and observe the planet (green) orbiting the Sun (yellow).

Draw the shape of the orbit and the location of the Sun relative to the orbit.

Based on the most precise observations available at the time—made by the astronomer Tycho Brahe—Kepler described the orbits of all planets in his first law.

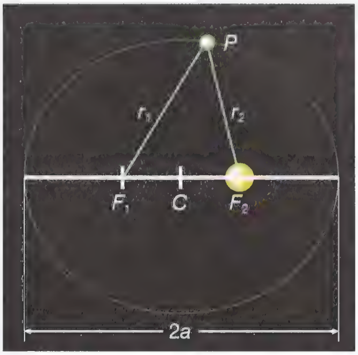
Kepler's first law, the law of elliptical orbits, stated that the orbit of a planet around the Sun is an ellipse, with the Sun located at one of the two focal points of the ellipse.

You might want to think of an ellipse as a circle that has been stretched about an **axis of symmetry**. The classical geometrical way of describing an ellipse will prove more useful in this lesson.

axis of symmetry: a line that divides a shape into two identical parts

Defining the Ellipse and Focal Points

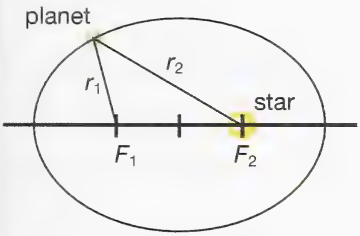
An ellipse is a curve on a plane. Each point, P, has a special relationship with two fixed points, F_1 and F_2. The sum of distances r_1 and r_2 from P to two fixed points, F_1 and F_2, is constant. The fixed points, F_1 and F_2, are the **focal points** of the ellipse. The sum of distances $r_1$and r_2 obeys the rule $r_1 + r_2 = 2a$, where $2a$ is the length of the major axis of the ellipse (the longest orbital *diameter* of the ellipse).



focal point: one of two special points used in determining the geometry of an ellipse

Self-Check

SC 2. Answer the following using the definitions above and the given values of r_1 and r_2 for the ellipse shown. Remember that 1 AU = 1.496×10^{11} m.



$r_1 = 1.736 \times 10^{11}$ m
$r_2 = 2.543 \times 10^{11}$ m

a. The longest mean orbital diameter (major axis) of the planet's orbit is ______________ m.

b. The mean orbital diameter expressed in astronomical units is ______________ AU.

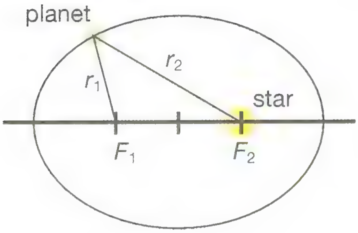
Check your work with the answer in the appendix.

Module 5: Lesson 3 Assignment

Remember to submit the answer to TR 2 to your teacher as part of your Lesson 3 Assignment in your Module 5 Assignment Booklet.

Try This

TR 2. Answer the following using the definitions above and the given values of r_1 and r_2 for the ellipse shown.



$r_1 = 1.396 \times 10^{11}$ m
$r_2 = 2.432 \times 10^{11}$ m

a. The longest mean orbital diameter (major axis) of the planet's orbit is ____________ m.
b. The mean orbital diameter expressed in astronomical units is ____________ AU.

Kepler's Second Law

Module 5: Lesson 3 Assignment

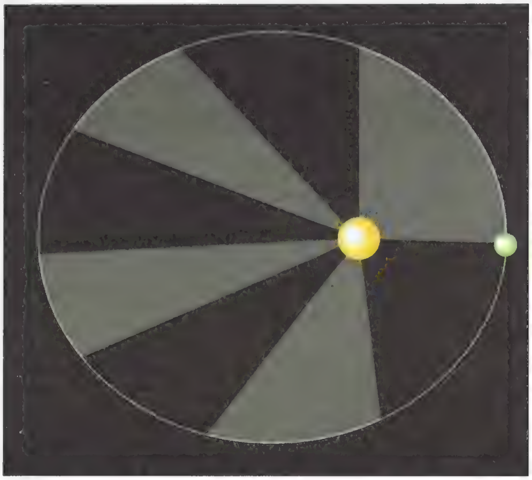
Remember to submit the answers to TR 3 and TR 4 to your teacher as part of your Lesson 3 Assignment in your Module 5 Assignment Booklet.

Try This

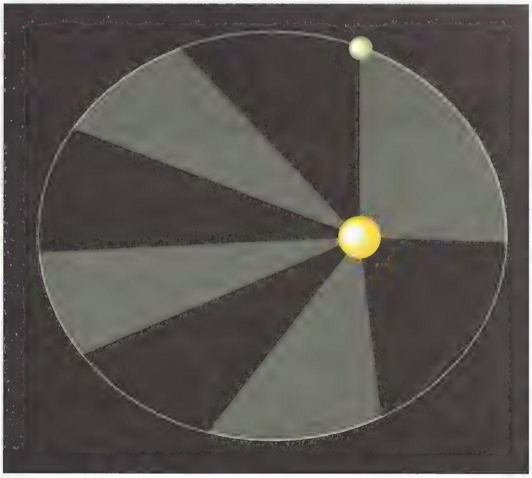
TR 3. Use the simulation to investigate Kepler's second law.

- Click the "Rewind" button ( ).
- Click the "Data" button ( ). Move the ellipse to the right so you can see it all by clicking in the ellipse and dragging it to the right.
- Click the "Sweep" button ( ). This will display sectors of equal area.
- Click the "Play" button ( ). "Pause" the planet ( ), and use the "Step forward" button ( ) to position the planet according to the following diagrams.

a. Record the time display below each figure.



$t_1 =$ __________



$t_2 =$ __________

b. Calculate the time interval for the planet to travel the sector ($t_2 - t_1$).

$$\Delta t = t_2 - t_1$$
$$= \_\_\_\_$$

TR 4. Using the same method as in TR 3, measure the times required for the planet to traverse at least three of these sectors of equal area. Compare the time values. What do you observe?

Using Brahe's observations, Kepler provided an explanation of the answer to TR 4 in his second law.

Kepler's second law, the law of equal areas, stated that in equal time intervals, the radius vector from the Sun to a planet sweeps across equal areas. (This means that planets move fastest when they are closest to the Sun and slowest when they are farthest from the Sun.)

Read

To reinforce what you have discovered, read about Kepler's first and second laws on pages 269 and 270 of the textbook.

Watch and Listen

Go to your Physics 20 Multimedia DVD, and watch the video called "Kepler's Discovery."

Kepler's Third Law

Module 5: Lesson 3 Assignment

Remember to submit the answer to TR 5 to your teacher as part of your Lesson 3 Assignment in your Module 5 Assignment Booklet.

Try This

TR 5. Use the simulation to investigate Kepler's third law.

- Click the "Rewind" button ( ).
- Click the "Sweep" button ( ). This will hide the sectors of equal area.
- Click the "Play" button ( ).

a. Record the length of the semi-major axis (the mean or average distance between the planet and the Sun or mean *orbital radius*) from the slider at the bottom of the window. The conventional representation for orbital radius is *r*.

$r =$ ____________ AU

b. Measure the period of the planet (the time to complete one revolution). You may want to click "Replay" ( ) in order to set the time to zero before making any measurements.

trial 1: Period: ___________ Year

c. Modify the eccentricity by adjusting the eccentricity slider. (**Note:** Do not adjust the semi-major axis.) Again, measure the period of the planet (time for one complete revolution).

trial 2: Period: ___________ Year

d. Did changing the orbital eccentricity change the period of the orbit?

Kepler noticed that the eccentricity *did not* change the period of the orbit. This fact is illustrated in his third law.

Kepler's third law, the law of periods, stated for all planet orbits, the square of the period is proportional to the cube of the semi-major axis (the mean distance from the Sun or mean orbital radius). Expressed as an equation it is

$$K = \frac{T^2}{r^3}$$

Quantity	Symbol	SI Unit	Common Units
orbital period	T	s	y or a
orbital radius (semi-major axis)	r	m	AU
Kepler's constant	K	$\frac{s^2}{m^3}$	$\frac{y^2}{AU^3}$

Read

Read about Kepler's third law beginning on page 271 and continuing to the middle of page 273 of the textbook.

Module 5: Lesson 3 Assignment

Remember to submit the answers to TR 6 and TR 7 to your teacher as part of your Lesson 3 Assignment in your Module 5 Assignment Booklet.

Try This

TR 6. Calculate Kepler's constant for the solar system using the recorded value for the semi-major axis length and the period from trial 1 or trial 2 in TR 5. (They should be the same.)

$K =$

TR 7. Verify Kepler's constant by changing the length of the semi-major axis to 1.8 AU and recording the orbital period by clicking "Play" on the simulation and watching it for a complete orbit. Use the "Pause" and "Step" buttons near the end of the full orbit to get the time accurately. Calculate the constant of proportionality again. Circle the units of the proportionality constant.

$r =$ ______________ AU

$T =$ ______________ Year

$K =$ ______________

Comment: In the units of AU and Year, the proportionality constant is numerically equal to 1. Notice that if T is in Years and r in AU, then the equation is as simple as $T^2 = r^3$.

Self-Check

SC 3. Solve question 1 of the “Practice Problems” on page 272 of the textbook.

Check your work with the answer in the appendix.

Module 5: Lesson 3 Assignment

Remember to submit the answer to TR 8 to your teacher as part of your Lesson 3 Assignment in your Module 5 Assignment Booklet.

Try This

TR 8. The orbital radius (semi-major axis) of the orbit of Ceres, an asteroid circling the Sun between Mars and Jupiter, is 2.77 AU. Using Kepler's constant for the solar system, calculate the orbital period of Ceres in years.

Read

The moons circling planets have a different constant, K, because their centres are different. To see how Kepler’s laws apply to moons, read from the middle of page 273 through page 275 of the textbook.

Self-Check

SC 4. Solve question 1 of “Practice Problems” on page 275 of your textbook.

Check your work with the answer in the appendix.

Module 5: Lesson 3 Assignment

Remember to submit the answer to TR 9 to your teacher as part of your Lesson 3 Assignment in your Module 5 Assignment Booklet.

Try This

TR 9. Suppose a new moon X was discovered orbiting Jupiter with an orbital period of 42 Earth days. What would be its expected mean orbital radius, using the data for Callisto in “Table 5.6” on page 274 of the physics textbook as a reference?

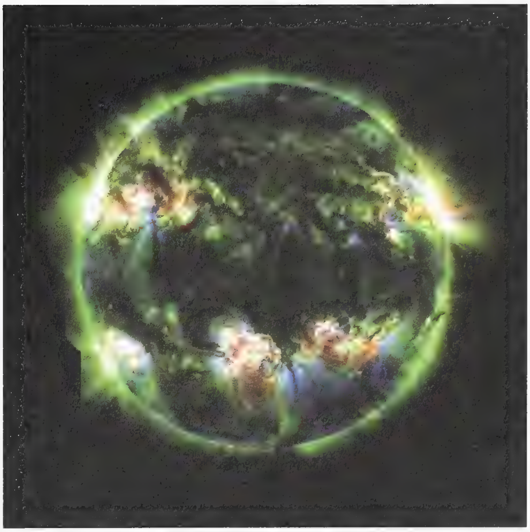
Reflect and Connect

© Olga Shelego/shutterstock

Astronomy continues to be a field that attracts many scientists, as does the exploration of space. Throughout history, there have been numerous inventions that either relied on astronomy or aided in the study of astronomy and space. Kepler's laws accurately describe the motion of the planets. Based on similar principles, other astronomical devices were designed based on the predictable motion of the heavens and Earth's motion within it. Consider the sundial that relies on the relative position of the Sun to keep accurate time. Another invention related to the periodic motion of the solar system was the astronomical clock (pictured on the left). This clock has special mechanisms to accurately display the relative position of the Sun, Earth, and Moon at any given time. The first astronomical clocks were produced 300 years before Kepler's time. They instantly gained appeal because they represented a philosophical position that the motion of celestial objects, such as the Moon, were ordered and part of a heavenly ordained universe. Since Galileo worked with the telescope in the seventeenth century, the telescope has undergone major changes and is now one source that provides scientists with high-quality, reliable data.

Here is an ultraviolet image of the Sun's active photosphere (the visible surface of the star), captured by the TRACE space telescope. This image contains details that were unimaginable in Kepler's time.

Leading-edge astronomy is no longer performed from a hilltop with an optical telescope.



Courtesy of TRACE, Stanford Institute, NASA

Discuss

In the discussion forum, post a general overview of one of the vast number of astronomical data-collection technologies currently being used. Describe the purpose and type of astronomical data that is being collected. Remember, the technology can be found on Earth's surface, in orbit beyond Earth's atmosphere, or in transit to places beyond the solar system.

You can use the following table to evaluate your overview.

	4	3	2	1	
Describing Technology	One technology is described in detail.	One technology is described, but the purpose or type of data being collected is weak.	One technology is described, but the purpose or type of data being collected is missing.	One technology is described briefly.	___
Information Gathering	Information is gathered from multiple sources and cited properly.	Information is gathered from multiple sources but not cited.	Information is gathered from limited sources and cited properly.	Information is gathered from limited sources and not cited.	___
Summary Paragraph	Paragraphs are well organized and demonstrate logical sequencing and sentence structure.	Paragraphs are well organized, but demonstrate illogical sequencing or sentence structure.	Paragraphs are well organized, but demonstrate illogical sequencing and sentence structure.	Paragraphs are weakly organized.	___
Total					___

Reflect on the Big Picture

Each of the Reflect on the Big Picture sections in this module will ask you to consider the movement of objects in a circle or part of a circle. Complete at least one of these reflection activities:

- Research and develop a timeline showing the major milestones in humanity's understanding of the paths of the planets.
- Research one of the early descriptions of planetary motion. (Be sure to identify the scientist who gave the description.) Based on what you have studied, can you identify one or two flaws in the description?

Store your completed reflection in your Physics 20 Course Folder.

Module 5: Lesson 3 Assignment

Make sure you have completed all of the questions for the Lesson 3 Assignment. Check with your teacher about whether you should submit your assignment now or wait until all of the Module 5 assignments have been completed.

Lesson Summary

In this lesson you focused on the solution to this question:

- How did Kepler's laws help calculate the position of the planets as they orbited the Sun?

Kepler's three laws describe the motion of the planets as they orbit the Sun.

Kepler's First Law

The orbit of a planet around the Sun is an ellipse, with the Sun located at one of the two focal points of the ellipse.

Kepler's Second Law

In equal time intervals, the radius vector from the Sun to a planet sweeps across equal areas.

Kepler's Third Law

For all planet orbits, the square of the period is proportional to the cube of the semi-major axis (the mean distance from the Sun or mean orbital radius). Expressed as an equation, it is

$$K = \frac{T^2}{r^3}$$

Lesson Glossary

astronomical unit: the average distance between Earth and the Sun

axis of symmetry: a line that divides a shape into two identical parts

calendar year: 365 days or 366 days (leap year)

elliptical: having the shape of an ellipse or oval

epicycle: a circle that rolls along the circumference of another circle

focal point: special points used in describing an ellipse

sidereal year: the orbital period of Earth

Lesson 4—Planetary and Satellite Motion

Get Focused

In Lesson 3 you learned that the motion of all planets in the solar system could be described by Kepler's three laws. What you did not cover was the explanation for the motion. The Moon, for example, orbits Earth along an elliptical path. Therefore, it would appear to approach and then recede as it orbits Earth. This is confirmed in the next Watch and Listen section, which shows a series of images taken as the Moon orbits Earth over a period of a month. As you can see, the motion is very **periodic**. Why does this happen? What forces are involved?

© William Attard McCarthy/shutterstock

periodic: recurring at regular intervals

Module 5: Lesson 4 Assignments

In this lesson you will complete the Lesson 4 Assignment in the Module 5 Assignment Booklet.

- LAB—LAB 1, LAB 2, LAB 3, LAB 4, LAB 5, and LAB 6
- Discuss

You must decide what to do with the questions that are not marked by the teacher.

Remember that these questions provide you with the practice and feedback that you need to successfully complete this course. You should respond to all the questions and place those answers in your course folder.

Watch and Listen

Go to your Physics 20 Multimedia DVD, and explore the segment called "Motions of the Moon." How did Sir Isaac Newton come to explain the motion of planets and other celestial bodies? As discussed in Module 2: Lesson 2, Newton's universal law of gravitation is involved in this explanation. The term *universal* means that the motions of the Moon, the planets, and artificial satellites (such as the space station) are all based on similar principles. At the same time, the orbit of any satellite or moon exhibits circular motion; therefore, principles of circular motion will have to be considered as well.

As you work through this lesson, keep this question in mind:

- How are planetary and satellite motion explained using universal gravitation and the principles of circular motion?

Explore

Understanding the Acceleration of a Satellite

To maintain periodic, circular motion—such as that of an orbiting satellite—an inward acceleration must act at all times. The nature and origin of this acceleration is found in Newton's universal law of gravitation. Recall the two definitions of weight derived in Module 4: Lesson 2: is as follows

$$W = mg \text{ and } W = \frac{G m_1 m_2}{r^2}$$

Equating these two definitions of weight will yield a general expression for the magnitude of the acceleration acting on a satellite in terms of

- the satellite's mass, m_1
- Earth's mass, m_2
- the distance, r, of the satellite from the centre of Earth

$$m_1 g = \frac{G m_1 m_2}{r^2}$$
$$g = \frac{G m_2}{r^2}$$

Use the simulation to explore the nature of the acceleration acting on a satellite at various distances from Earth's surface.

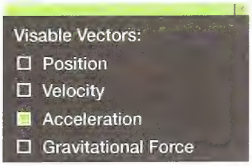
trajectory: the path of a moving body through space

Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "Weight and Orbits" in the search bar. Choose the item called "Weight and Orbits (Grade 11)." This applet simulates the trajectory weight and orbits and gravitational force acting on a particle in Earth's gravitational field by varying its initial position and velocity. You can learn more about the simulation and how to use it by reading Show Me found at the top of the simulation screen.

Self-Check

SC 1. The distance between the centre of Earth and the satellite (r) is 6.37×10^6 m + 6.00×10^5 m = 6.97×10^6 m. Using the equations above, calculate the acceleration of a satellite that is orbiting Earth at an altitude of 600 km. Use the simulation to verify your answer. (Set the x position to 0 km and the y position to 600 km; then press "Enter." Click "Data," and look for the acceleration value in the data box.)

SC 2. What happens to the magnitude of the acceleration as the satellite moves farther away from Earth? Review this by clicking the "Display Vectors Selection Panel" button (Vectors) and turning on the acceleration vector and then de-selecting the velocity vector.



Then click off the "Display Vectors Selection Panel." Change the scale to 100 px = 5000 km to get a better view. Drag the satellite to various positions around Earth. What do you notice about the magnitude and direction of the acceleration vector?

SC 3. On the simulation, set the scale to 100 pix = 10 km. This is the highest zoom setting. (The satellite will be outside of the bounds of the window.) Set $x = 0$ km and $y = 0$ km to move the satellite back to the North Pole. Now drag the satellite around the display area, and observe the changes in the magnitude of the acceleration vector. Are the changes in magnitude larger or smaller near Earth's surface?

Check your work with the answer in the appendix.

How can the fact that there is so little change be explained? At this scale setting, the changes in distance that are possible within the window are only tiny fractions of Earth's radius. Significant changes in the weight only occur when the distance from Earth's surface changes by a considerable amount. In Newton's universal law of gravitation, the distance of separation, r, is always measured from the centre of the mass. Therefore, Earth's radius must always be added to a satellite's altitude when applying Newton's equations to describe the motion.

Lesson 4 Lab: Acceleration, Weight, and Velocity of Satellites

Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "Weight and Orbits" in the search bar. Choose the item called "Weight and Orbits (Grade 11)." You will continue to use this simulation in this lab.

Problem 1

Do the weight and the acceleration of a satellite change during the satellite's motion? If so, how do they change?

Module 5: Lesson 4 Assignment

Remember to submit the answers to LAB 1 and LAB 2 to your teacher as part of your Lesson 4 Assignment in your Module 5 Assignment Booklet.

Procedure 1

LAB 1. Complete the following steps:

- Reset the simulation.
- Select the "Trace" ( ), "Data" ( ), and "Grid" ( ) buttons.
- Zoom out by setting the "Scale" to 100 px = 100 000 km so you can see Earth in its entirety.
- Use the "Display Vectors Selection Panel" button ( ) to select the acceleration and weight (gravitational force) vectors, and then click off the "Display Vectors Selection Panel."
- Play the motion. Observe the weight vector during the motion, as well as the weight value displayed in the data box. Press "Pause" at various points, if necessary, to look at the data.

a. Does either the weight vector or the weight value change?

b. Describe what happens to the direction of the weight vector as the satellite orbits Earth.

c. Describe what happens to the magnitude of the weight vector as the satellite orbits Earth. If it changes, how can this be explained using Newton's equations for weight?

Notice that as the satellite orbits, the weight and acceleration vectors vary in the same way. The satellite is not always the same distance from Earth's centre. This causes the acceleration and, by extension, the weight of the satellite to change.

LAB 2. Is the satellite ever "weightless" in any orbit? Experiment with different orbits in different scale settings, and move the satellite as far away as possible. Explain your observations in terms of the equations that describe weight.

Problem 2

How can you calculate the velocity of a satellite (m_1) moving in a perfectly circular orbit?

Module 5: Lesson 4 Assignment

Remember to submit the answers to LAB 3, LAB 4, and LAB 5 to your teacher as part of your Lesson 4 Assignment in your Module 5 Assignment Booklet.

Procedure 2

LAB 3. Complete the following steps:

- Reset the simulation.
- Use the "Display Vectors Selection Panel" button ( ) to select the magenta velocity vector and the green force of gravity vector, and then click off the "Display Vectors Selection Panel."
- Display the data box in order to observe the satellite's weight. While the satellite is stationary, vary the satellite's initial velocity by dragging the tip of the velocity vector.

Do you see any changes in the force of gravity, the acceleration due to gravity acting on the satellite, or the value of the weight shown in the data box? Explain your observations in terms of Newton's universal law of gravitation. (Specifically, does the force of gravity depend on the velocities of the interacting objects?)

LAB 4. Find out if a projectile can become a satellite, given an appropriate initial velocity and position.

- Switch the scale setting to 100 pix = 5000 km.

- Bring the satellite down to the surface by setting the *x* and *y* positions to 0 km, respectively; then drag the satellite up slightly.

- Turn on the velocity vector, and set it to 3 500 m/s in a horizontal direction ($\theta = 0°$).

- Press "Play," and observe how far around the planet the satellite makes it before crashing into the surface.

- Try a variety of velocities until you are able to have the satellite orbit the planet successfully within the field of view.

Based on your observations, fill in the blanks with these terms: *large*, *small*, *parallel*, *perpendicular*.

A projectile will become a satellite in a nearly circular orbit when the magnitude of the velocity is ____________ and it is directed ___________ to Earth's surface.

LAB 5. Using a trial-and-error method, try to find the exact horizontal speed that a satellite would need to have at an altitude of 1000 km to make a circular orbit.

circular orbit speed = _________________

Read

There is an easier way to determine the speed of a satellite. Read pages 276 to 278 of your textbook, starting at "Newton's Version of Kepler's Third Law."

How can you calculate the velocity of a satellite (m_1) moving in a perfectly circular orbit? Since the gravitational force is an inward force, it is possible to generate an expression for the velocity of a satellite in a perfectly circular orbit. (If you're wondering why we chose a circular orbit, it's because the math is a lot easier than for other types of orbit.)

$$F_{\text{inward}} = F_g$$

$$\frac{m_1 v^2}{r} = \frac{G m_1 m_2}{r^2}$$

$$v^2 = \frac{G m_2}{r}$$

$$v = \sqrt{\frac{G m_2}{r}}$$

Self-Check

SC 4. Answer question 1 of "Practice Problems" on page 278 of the textbook.

Check your work with the answer in the appendix.

Read

Often the position of a satellite is given as altitude above Earth instead of as an orbital radius. Read pages 279 and 280 of your textbook. It is similar to how you have worked with altitude in the Weight and Orbits simulation.

Self-Check

SC 5. Answer question 1 of "Practice Problems" on page 279 of the textbook.

Check your work with the answer in the appendix.

Module 5: Lesson 4 Assignment

Remember to submit the answer to LAB 6 to your teacher as part of your Lesson 4 Assignment in your Module 5 Assignment Booklet.

LAB 6. Use the equation derived above to calculate the exact value of the initial speed that will produce a circular orbit of the satellite at an altitude of 1000 km above Earth. Verify your answer using the simulation, and compare it to your answer in LAB 5.

Conclusion

Newton's universal law of gravitation and the principles of circular motion can be applied to accurately explain the observed motion of planets and satellites.

Equating the inward force and the universal law of gravitation gives an expression for the velocity of an orbiting planet or satellite.

Equating Newton's second law and the universal law of gravitation gives an expression for the inward acceleration of an orbiting planet or satellite.

By determining the velocity and the acceleration of an orbiting planet or satellite, it is possible to explain the observed orbital motion, which is accurately described by Kepler's three laws.

Read

Newton's law of universal gravitation can give us more information about celestial objects besides their speed in orbit. Read pages 280 to 283 of your textbook, starting at "Determining the Mass of a Celestial Body."

Self-Check

SC 6. Answer question 13 from "5.3 Check and Reflect" on page 286 of the textbook.

Check your work with the answer in the appendix.

Read

Newton's law of universal gravitation has enabled humankind to successfully launch a myriad of artificial satellites. Read pages 284 to 286 of your textbook, starting at "Artificial Satellites."

Reflect and Connect

Newton not only provided the explanation of planetary motion, his explanation can be extended to explain the motion of other celestial bodies. The Moon's orbit is accurately explained by considering Newton's universal law of gravitation. It is the gravitational force between Earth and the Moon that produces an inward acceleration, constantly changing the direction of the Moon's velocity and making it follow a circular path. Given the mean orbital radius (average distance between the centres of mass) and the mass of Earth, it is possible to determine both the velocity and acceleration of the Moon. Completing a similar analysis for the Chandra X-ray Observatory—an artificial satellite—shows that both the Moon and Chandra obey the same universal law that explains their motion.

	Moon (Natural Satellite) $r = 384\ 400$ **km**	**Chandra X-ray Telescope (Artificial Satellite)** $r = 120\ 963$ **km**
Acceleration	$g = \frac{G m_2}{r^2}$ $g = \frac{(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2)(5.98\times10^{24}\,\text{kg})}{(3.844\times10^{8}\,\text{m})^2}$ $g = \frac{6.67\times5.98}{(3.844)^2}\times10^{-3}\ \text{m/s}^2$ $g \doteq 2.699\ 356\ 593\times10^{-3}\ \text{m/s}^2$ $g = 2.70\times10^{-3}\,\text{m/s}^2$ (correct to 3 significant digits)	$g = \frac{G m_2}{r^2}$ $g = \frac{(6.67\times10^{-11}\,\text{N}\bullet\text{m}^2/\text{kg}^2)(5.98\times10^{24}\,\text{kg})}{(1.20963\times10^{8}\,\text{m})^2}$ $g = \frac{6.67\times5.98}{(1.20963)^2}\times10^{-3}\ \text{m/s}^2$ $g \doteq 27.259\ 753\ 19\times10^{-3}\ \text{m/s}^2$ $g = 2.73\times10^{-2}\,\text{m/s}^2$ (correct to 3 significant digits)

Velocity	$v=\sqrt{\dfrac{Gm_2}{r}}$ $v=\sqrt{\dfrac{(6.67\times10^{-11}\,\text{N•m}^2/\text{kg}^2)(5.98\times10^{24}\,\text{kg})}{(3.844\times10^{8}\,\text{m})}}$ $v=\sqrt{\dfrac{6.67\times5.98}{3.844}}\times10^{5}\ \text{m/s}$ $v\doteq1018.642\,565\ \text{m/s}$ $v=1.02\times10^{3}\ \text{m/s}$ (correct to 3 significant digits)	$v=\sqrt{\dfrac{Gm_2}{r}}$ $v=\sqrt{\dfrac{(6.67\times10^{-11}\,\text{N•m}^2/\text{kg}^2)(5.98\times10^{24}\,\text{kg})}{(1.20963\times10^{8}\,\text{m})}}$ $v=\sqrt{\dfrac{6.67\times5.98}{1.20963}}\times10^{5}\ \text{m/s}$ $v\doteq1815.880\,372\ \text{m/s}$ $v=1.82\times10^{3}\ \text{m/s}$ (correct to 3 significant digits)

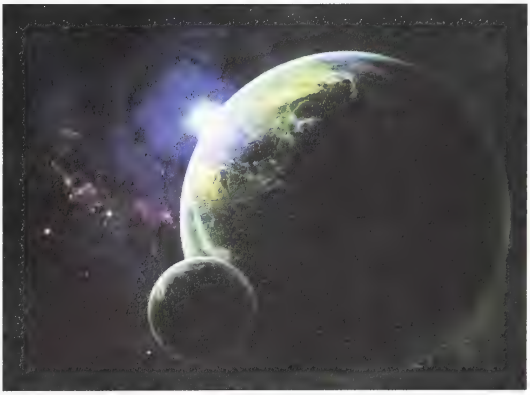
Comparing the radius values used in Newton's universal law of gravitation, the Chandra X-ray Observatory is about three times closer to Earth than the Moon. This is reflected in the fact that the observatory travels at a greater rate of speed with a larger acceleration to keep it on a circular path. Both motions are explained in exactly the same terms.

Module 5: Lesson 4 Assignment

Remember to submit the answer to Discuss to your teacher as part of your Lesson 4 Assignment in your Module 5 Assignment Booklet.

Discuss

In 1995, the first planet outside of our solar system was discovered and named "51 Pegasi b" after the star it orbits. Planets like this are called extrasolar because they are outside Earth's solar system. Compared to the bright stars they orbit, extrasolar planets are very difficult to directly observe, given the astronomical distance they are from Earth. One of the most common and successful ways to find a planet is to observe the star it orbits.



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Using the article titled "Then, Now, and Future: Extrasolar Planets" on page 283 of the textbook as a reference, propose an indirect method for finding extrasolar planets using the principles of circular motion and Newton's universal law of gravitation. If such planets were discovered with this method, explain why it would be unlikely that life would exist on them.

Reflect on the Big Picture

Each of the Reflect on the Big Picture sections in this module will ask you to consider the movement of objects in a circle or part of a circle. You may want to combine your reflection for this lesson with some of the work on the project.

- Placing artificial satellites in orbit around Earth has become commonplace over the last couple of generations. Research the orbits of the GPS satellites and the required speed of the satellites. Write a short report about your findings.
- Prepare a computer simulation of the orbits of GPS satellites.
- Calculate the speed of both of Mars' moons and their distance from Mars.

Store your completed reflection in your Physics 20 course folder.

Module 5: Lesson 4 Assignment

Make sure you have completed all of the questions for the Lesson 4 Assignment. Check with your teacher about whether you should submit your assignment now or wait until all of the Module 5 assignments have been completed.

Lesson Summary

In this lesson you focused on the solution to this question:

- How are planetary and satellite motion explained using universal gravitation and the principles of circular motion?

Newton's universal law of gravitation and the principles of circular motion can be applied to explain the observed motion of planets and satellites.

Equating the inward force and the universal law of gravitation gives an expression for the velocity of an orbiting planet or satellite.

$$F_{\text{inward}} = F_{\text{g}}$$

$$\frac{m_1 v^2}{\text{r}} = \frac{G m_1 m_2}{r^2}$$

$$v = \sqrt{\frac{G m_2}{r}}$$

Equating Newton's second law and the universal law of gravitation gives an expression for the inward acceleration of an orbiting planet or satellite.

$$F_{\text{inward}} = F_g$$

$$m_1 a = \frac{G m_1 m_2}{r^2}$$

$$a = \frac{G m_2}{r^2}$$

By determining the velocity and the acceleration of an orbiting planet or satellite, it is possible to explain the observed orbital motion, which is accurately described by Kepler's three laws.

Lesson Glossary

periodic: recurring at regular intervals

trajectory: the path of a moving body through space

Module 5—Circular Motion

Module Summary

When an object travels in a circular path, there is an inward force that causes the direction of the motion to change. The inward force can be friction, tension, gravitational, and so on. The magnitude of the force is related to the mass of the object, radius of the circle, and speed of the object.

Inward (centripetal) force can be mathematically described by the relationship between the velocity, mass, and radius of an object rotating in a horizontal plane. Expressed as an equation, it is

$$F_{\text{inward}} = \frac{mv^2}{r}$$

Quantity	Symbol	SI Unit
inward (centripetal) force	F_{inward}	N
mass	m	kg
radius of circular path	r	m
speed	v	m/s

Applying Newton's second law to the equation for centripetal force gives the following expression for centripetal acceleration:

$$ma = \frac{mv^2}{r}$$
$$a_c = \frac{v^2}{r}$$

The situation becomes more complicated when gravity acts on an object that follows a vertical circular path. You can use free-body analysis to understand the net inward force and tension acting on such an object.

At the top of the vertical circle, the tension acting is described by

$$T = \frac{mv^2}{r} + (-mg),\ \text{where } g = +9.81\ \text{m/s}^2$$

If the tension reaches zero at the top of the arc, the object is in a momentary state of free fall. The speed at which this will occur is described by

$$0 = \frac{mv^2}{r} + (-mg)$$

$$\frac{\cancel{m}v^2}{r} = \cancel{m}g$$

$$v = \sqrt{gr}$$

At the bottom of the vertical circle, the tension acting is greater because it opposes the force due to gravity.

$T = \frac{mv^2}{r} + mg$, where $g = +9.81\ \text{m/s}^2$

From this analysis, you will be able to determine the acceleration acting on an object or rider at any point in the vertical circle, leading to a better understanding of the causes of the sensations experienced when moving in vertical circles.

Kepler's three laws describe the motion of the planets as they orbit the Sun.

- Kepler's first law stated the orbit of a planet around the Sun is an ellipse, with the Sun located at one of the two focal points of the ellipse.
- Kepler's second law stated that in equal time intervals, the radius vector from the Sun to a planet sweeps across equal areas.
- Kepler's third law stated that for all planet orbits, the square of the period is proportional to the cube of the semi-major axis (the mean distance from the Sun or mean orbital radius). Expressed as an equation, it is

$$K = \frac{T^2}{r^3}$$

Newton's universal law of gravitation and the principles of circular motion can be applied to explain the observed motion of planets and satellites.

Equating the inward force and the universal law of gravitation gives an expression for the velocity of an orbiting planet or satellite.

$$F_{\text{inward}} = F_g$$

$$\frac{m_1 v^2}{r} = \frac{Gm_1 m_2}{r^2}$$

$$v^2 = \frac{Gm_2}{r}$$

$$v = \sqrt{\frac{Gm_2}{r}}$$

Equating Newton's second law and the universal law of gravitation gives an expression for the inward acceleration of an orbiting planet or satellite.

$$m_1 g = \frac{G m_1 m_2}{r^2} \qquad m_1 g = \frac{G m_1 m_2}{r^2}$$

$$g = \sqrt{\frac{G m_2}{r^2}} \qquad g = \frac{G m_2}{r^2}$$

By determining the velocity and the acceleration of an orbiting planet or satellite, it is possible to explain the observed orbital motion, which is accurately described by Kepler's three laws.

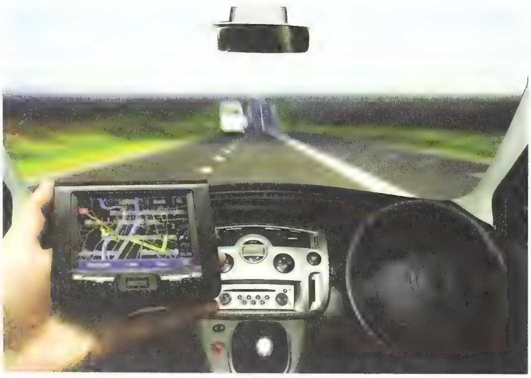
Module 5 Assessment

The assessment for Module 5 consists of four (4) assignments, as well as a final module inquiry project.

- Module 5: Lesson 1 Assignment
- Module 5: Lesson 2 Assignment
- Module 5: Lesson 3 Assignment
- Module 5: Lesson 4 Assignment
- Module 5 Project

Module 5 Project: Global Positioning Satellites

This module project is based on an understanding of global positioning systems (GPS) and how they relate to circular motion, gravitational fields, and the past and future impacts of technology on society.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Global positioning systems are proliferating in our society in many ways. Cell phones, cars, pet tags, and other applications are evolving to incorporate this technology. Applications, such as "arrive alive," will allow cell-phone position, movements, and whereabouts to be tracked in real time. Imagine parents monitoring where their children are at all times, being notified if the phone travels outside a "boundary" or if the motion of the phone is in a vehicle that exceeds the local speed limit. These are issues that are being discussed now, and they are all related to satellite technology that is based on circular motion and gravitational fields. In fact, if you have a cell phone, you are already carrying this technology.

Project Tasks

This inquiry project involves four major questions with two related tasks in each.

1. How is GPS technology used to solve problems?

    a. Investigate where GPS applications are being used, noting societal issues that have "popped-up" as a result of their applications.

b. Create a list of applications or problems that GPS technology could be used to solve.

2. How does GPS work?

    a. Research the network of satellites that are used to power the current world GPS system, and describe how this technology functions.

    b. Report on the orbital specifications of the GPS satellites. For example, report about the mass, altitude, and orbital period of GPS satellites.

3. Describe how principles of circular motion determine the orbital path of a satellite.

    a. Given a satellite's mass and orbital altitude, how is the speed of the satellite determined?

    b. How is a satellite's speed and altitude related to the launch vehicle requirements?

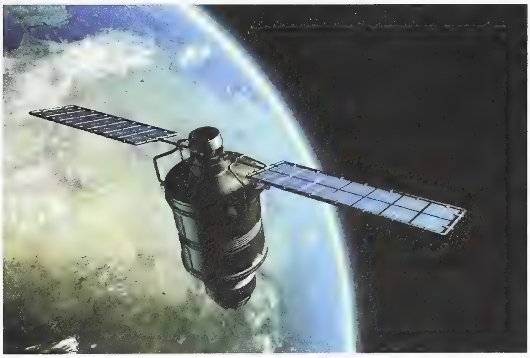
4. How does Earth's gravitational field cause the orbital path of the satellite?

    a. Determine the acceleration due to gravity at the GPS satellite's orbital altitude using Newton's universal law of gravitation.

    b. Relate the orbital acceleration to the velocity as a verification using $F_{inward} = F_g$.

Submitting Your Project

Your project must be submitted as a computer presentation using the software of your choice (e.g., PowerPoint, Word, HTML, PDF).



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Your presentation will include a simulation of the GPS orbital path that verifies your calculations. You may use the weight and orbits simulation used in Lesson 4, or you may create this simulation using other technologies. If you would like to use the simulation, go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms “Weight and Orbits” in the search bar. Choose the item called “Weight and Orbits (Grade 11).”

Your presentation will include answers to the four major questions posed under Project Tasks.

Your presentation should conclude by explaining how orbital knowledge supports the infrastructure for new technologies and their application in solving both old and new problems.

Be sure to include a reference page listing all of your research sources for this project. List all web pages, books, and magazine articles you use.

Research

Begin your research by searching the Internet for the phrase "Global Positioning System GPS."

You might also want to search for a tutorial on GPS.

How Will Your Project be Assessed?

You will be marked according to the following guidelines. You will be graded for your work answering the four major questions listed under Project Tasks.

1. How is GPS technology used to solve problems?			
1: Poor	**2: Satisfactory**	**3: Good**	**4: Excellent**
The project suggests applications that are unrelated to the technology or are not new.	The project suggests only a few limited applications, some not new.	The project explains multiple applications of GPS and suggests new and unique applications.	The project applies GPS to new applications and begins to identify risks and benefits.

2. How does GPS work?			
1: Poor	**2: Satisfactory**	**3: Good**	**4: Excellent**
The project demonstrates a lack of research and understanding of the technology.	The project demonstrates a basic understanding of the technology with some research.	The project demonstrates a solid understanding of the technology with relevant research.	The project demonstrates a solid understanding of the technology with exemplary research.

3. Describe how principles of circular motion determine the orbital path of a satellite.			
1: Poor	**2: Satisfactory**	**3: Good**	**4: Excellent**
The project does not apply relevant physics concepts.	The project applies basic equations to describe one aspect of circular motion.	The project applies all equations with some errors and misconceptions.	The project applies all equations to completely describe the motion of a satellite.

4. How does Earth's gravitational field cause the orbital path of the satellite?			
1: Poor	**2: Satisfactory**	**3: Good**	**4: Excellent**
The project does not apply Newton's laws to the motion.	The project applies one of Newton's laws for a basic explanation of the orbit.	The project applies all of Newton's laws correctly to explain the motion of the satellite.	The project applies and shows manipulation and substitution of Newton's laws to explain the motion of the satellite.

You will also be graded for the presentation and delivery of your Module 5 Project.

1: Poor	2: Satisfactory	3: Good	4: Excellent
The project presents content in a text-based document only.	The project presents content in multimedia form with limited discussion and explanation.	The project presents content in multimedia form with discussion and use of a simulation.	The project presents content in multimedia form with discussions, supporting documents, a simulation, and suggestions for further study and application.

Total available marks = 20 marks x 3 (weighting) = 60 marks

Module 5 Glossary

astronomical unit: the average distance between Earth and the Sun

axis of symmetry: a line that divides a shape into two identical parts

calendar year: 365 days or 366 days (leap year)

centripetal: directed toward the centre of a circle

elliptical: having the shape of an ellipse or oval

epicycle: a circle that rolls along the circumference of another circle

focal point: one of two special points used in describing an ellipse

frequency: the number of cycles in a time period (f)

horizontal plane: a plane perpendicular to a radius of Earth; usually used to suggest that there is no vertical component to motion or forces

period: the time for one complete cycle (T)

periodic: recurring at regular intervals

sidereal year: the orbital period of Earth

tension: a stress that tends to stretch an object

trajectory: the path of a moving body through space

uniform circular motion: motion of an object with a constant speed along a circular path

vertical circular motion: motion in a circular path where one diameter of the circle is vertical

weightless: experiencing little apparent gravitational pull

Self-Check Answers

Lesson 1

SC 1. When the ball was released, it travelled in the direction in which it was moving the moment the string was cut. This occurs because the string is no longer pulling on the ball and Newton's first law comes into play.

SC 2. B

SC 3.

Variables

Choose speed as the manipulated variable and force as the responding variable. The controlled variables are the length of the string, the mass of the ball, and the plane of the rotation.

Procedure

- Set up a data table with four columns. Label the first two to record the speed and the corresponding force.
- Start with a speed of 1.0 m/s. Run the simulation, and record the force required.
- Repeat: Add 3 m/s to the value of the previous speed every time.
- Graph the results with speed on the horizontal axis.
- If the graph does not show direct proportionality (a straight diagonal line), use the other columns of the data table to fill in v^2 *or* $\sqrt{v}$. Then graph that value against the force until you find a graph that gives the straight diagonal graph line indicating direct proportionality.

Conclusion

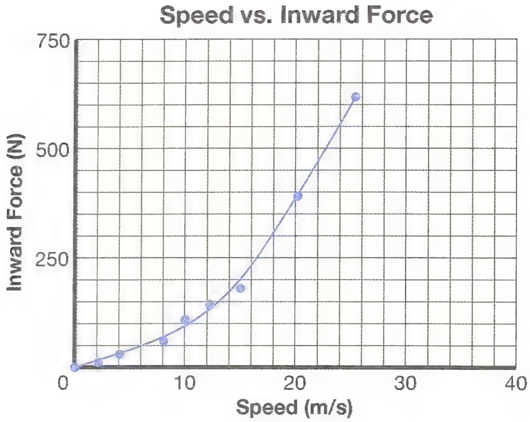
Express the relationship as a mathematical proportionality.

SC 4. The meaning of the word *centripetal* from its Latin roots means "centre seeking."

SC 5.

Speed (m/s)	Inward Force (tension) (N)		Speed (m/s)	Inward Force (tension) (N)		Speed (m/s)	Inward Force (tension) (N)
1.0	**1.0**		8.0	**64.0**		15.0	**225.0**
2.0	**4.0**		10.0	**100.0**		20.0	**400.0**
4.0	**16.0**		12.0	**144.0**		25.0	**625.0**

SC 6.



SC 7. $\vec{F}_c \propto v^2$

SC 8.

a. The acceleration vector is always directed towards the centre of the circle.

b. The velocity vector is always perpendicular to the acceleration vector (tangent to the circle) and oriented in the direction of rotation.

SC 9.

Given

rotation rate = 6.0×10^4 rpm

Required

the frequency (f) and period (T)

Analysis and Solution

Convert the rotation rate to frequency in hertz using a dimensional analysis ratio, and then convert the frequency to period.

$$6.0 \times 10^4 \ \frac{\text{revolutions}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 1.0 \times 10^3 \ \frac{\text{revolutions}}{\text{sec}}$$

This is a frequency of 1.0×10^3 Hz because a cycle per second is a hertz.

$$
\begin{aligned}
T &= \frac{1}{f} \\
&= \frac{1}{1.0 \times 10^{3}\ \text{Hz}} \\
&= \frac{1}{1.0 \times 10^{3}\ /\text{s}} \\
&= 1.0 \times 10^{-3}\ \text{s}
\end{aligned}
$$

Paraphrase

The frequency of rotation of the centrifuge is 1.0×10^3 Hz, and the period of rotation is 1.0 × 10–3 s.

SC 10.

Given

$v = 261.0$ km/h $\qquad r = 0.350$ m

Required

the period (T)

Analysis and Solution

Convert the speed to m/s, and then rearrange the formula $v = \frac{2\pi r}{T}$ to solve for the period. Substitute in the values and find the answer.

$$261.0\ \frac{\text{km}}{\text{h}} \times \frac{1\ \text{h}}{3600\ \text{s}} \times \frac{1000\ \text{m}}{1\ \text{km}} = 72.50\ \frac{\text{m}}{\text{s}}$$

$$
\begin{aligned}
v &= \frac{2\pi r}{T} \\
T &= \frac{2\pi r}{v} \\
&= \frac{2\pi(0.350\ \text{m})}{72.50\ \text{m/s}} \\
&\ 0.0303\ \text{s}
\end{aligned}
$$

Paraphrase

The period of rotation of the tire is 0.0303 s.

SC 11.

Given

diameter = 28.0 cm

$T = 0.110$ s

Required

the centripetal acceleration (a_c)

Analysis and Solution

The magnitude of the acceleration can be calculated using the formula $a_c = \frac{v^2}{r}$. The direction of centripetal acceleration is always towards the axis of rotation.

The radius is half the diameter, so $r = \frac{28.0\text{ cm}}{2} \times \frac{1\text{ m}}{100\text{ cm}} = 0.140$ m.

Determine the speed of the outermost edge of the Frisbee:

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \frac{2\pi(0.140\text{ m})}{0.110\text{ s}} \\ &= 7.997\text{ m/s} \end{aligned}$$

Use this value to calculate the centripetal acceleration:

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ a_c &= \frac{(7.997\text{ m/s})^2}{0.140\text{ m}} \\ &= 457\text{ m/s}^2 \end{aligned}$$

Paraphrase

The centripetal acceleration of the outer edge of the Frisbee is 457 m/s^2 directed towards the centre of the Frisbee.

SC 12.

Given

$m = 0.0021$ kg
$r = 23.0$ cm
$F_c = 0.660$ N

Required

the speed of the wheel (v)

Analysis and Solution

Rearrange the centripetal force formula to find the speed.

$$F_c = \frac{mv^2}{r}$$

$$v^2 = \frac{F_c r}{m}$$

$$\begin{aligned} v &= \sqrt{\frac{F_c r}{m}} \\ &= \sqrt{\frac{(0.660\ \text{N})(0.230\ \text{m})}{0.0021\ \text{kg}}} \\ &= 8.5\ \text{m/s} \end{aligned}$$

Paraphrase

The speed of the wheel is 8.5 m/s.

SC 13.

The centripetal force is proportional to the square of the speed:

$$F_c \propto v^2$$

If you triple the speed, you increase the centripetal force needed, the tension in the string, by 3^2 or nine times:

$$9F_c \propto (3v)^2$$

The inward force would increase by a factor of 9.

SC 14.

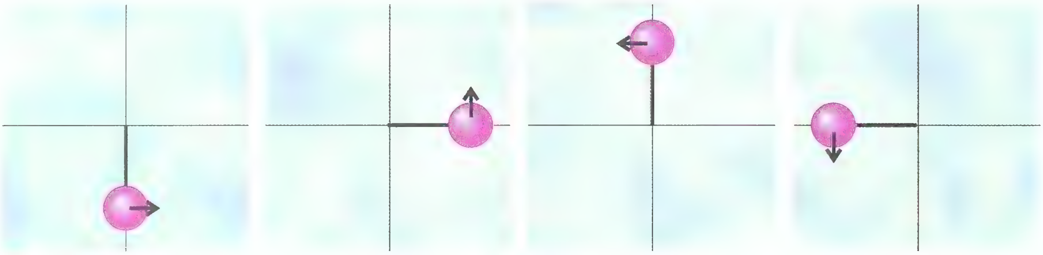
(a) friction

(b) tension in the rope

(c) gravitational force of Earth on the Moon

Lesson 2

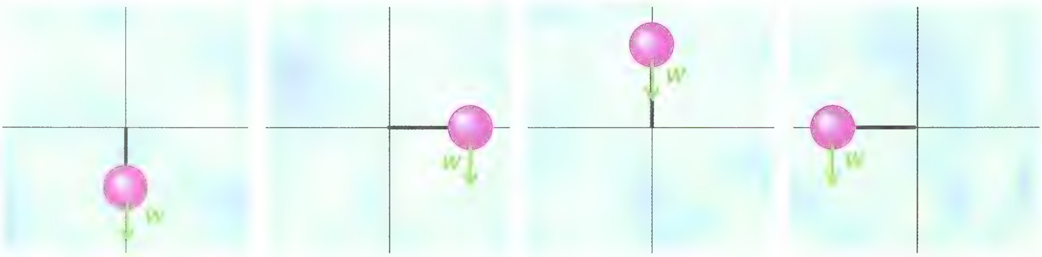
SC 1.



SC 2. The direction is constantly changing but remains perpendicular to the centre of the circle. The magnitude is greatest at the bottom of the circle and least at the top.

SC 3.

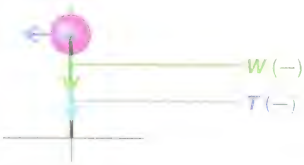
a.



b. The weight (gravitational force) is always directed **down**.

c. The magnitude and direction of the weight vector does not **change**.

SC 4.



SC 5.

a. 8.094 m/s.

b.

i. $T = \left(\frac{m v^2}{r}\right) + (-m g)$

ii. $T = \left(\frac{(2.0 \text{ kg})(8.094 \text{ m/s})^2}{(2.0 \text{ m})}\right) + \left(-(2.0 \text{ kg})\left(9.81 \text{ m/s}^2\right)\right) = 46\,N$

iii. tension measurement: **45.9** N

SC 6.

a. $v = \sqrt{gr}$

$= \sqrt{\left(9.81 \text{ m/s}^2\right)(3.00 \text{ m})}$

$= 5.42 \text{ m/s}$

b. initial velocity: **12.1** m/s

c. According to the tension measurements, when the bar changes to a red colour, the tension has become negative (reversed directions). This symbolizes that the ball would be in free fall if it were attached to a rope instead of a bar.

SC 7.

Given

$v = 20.0$ m/s

Required

the maximum radius of the loop (r)

Analysis and Solution

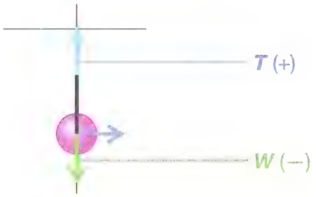
For the car to remain on the track at the top of the loop without hanging by its wheels, the centripetal force must be at least equal to the force of gravity. Equate the two forces and solve for the radius (r).

$$F_c = F_g$$
$$\frac{mv^2}{r} = mg$$
$$r = \frac{mv^2}{mg}$$
$$= \frac{v^2}{g}$$
$$= \frac{(20.0\ \text{m/s})^2}{9.81\ \text{m/s}^2}$$
$$= 40.8\ \text{m}$$

Paraphrase

The maximum radius of the loop for the car to go around safely is 40.8 m.

SC 8.



SC 9.

a. $F_{\text{inward}} = W + T$

b. positive

c. $(+F_{\text{inward}}) = (-W) + (+T)$

d. $T = (+F_{\text{inward}}) + (+W)$

e. $T = \left(\frac{mv^2}{r}\right) + (mg)$

SC 10.

a. speed = **12. 0** m/s

b.

i. $T = \left(\frac{mv^2}{r}\right) + (mg)$

ii. $$T = \left(\frac{(2.0\ \text{kg})(12.0\ \text{m/s})^2}{2.0\ \text{m}}\right) + (2.0\ \text{kg})(9.81\ \text{m/s}^2)$$
$$= 1.6 \times 10^2\ \text{N}$$

iii. tension measurement: **163.62** N

SC 11.

Given

$r = 30.0$ m $\qquad a = 9.81$ m/s^2

Required

the frequency of rotation of the space station (f)

Analysis and Solution

The centripetal acceleration of the space station must equal the acceleration due to gravity. Rearrange the centripetal acceleration formula to solve for frequency.

$$a_c = 4\pi^2 r f^2$$
$$f^2 = \frac{a_c}{4\pi^2 r}$$
$$f = \sqrt{\frac{a_c}{4\pi^2 r}}$$
$$= \sqrt{\frac{9.81\ \text{m/s}^2}{4\pi^2 (30.0\ \text{m})}}$$
$$= 9.10 \times 10^{-2}\ \text{Hz}$$

Paraphrase

The frequency of rotation of the space station must be 9.10×10^{-2} Hz.

Lesson 3

SC 1.

$$\text{speed } (v) = \frac{\text{distance } (r)}{\text{time } (t)} = \frac{\text{AU}}{\text{years}}$$

$$\text{acceleration } (\vec{a}) = \frac{\text{change in velocity } (\Delta\vec{v})}{\text{time } (t)} = \frac{\text{AU}}{\text{year}^2}$$

SC 2.

a. $2a = r_1 + r_2$

$= 1.736 \times 10^{11}\ \text{m} + 2.543 \times 10^{11}\ \text{m}$

$= 4.279 \times 10^{11}\ \text{m}$

The longest mean orbital diameter (major axis) of the planet's orbit is 4.279×10^{11} m.

b. $4.279 \times 10^{11}\ \text{m} \times \dfrac{1\ \text{AU}}{1.496 \times 10^{11}\ \text{m}} = 2.860\ \text{AU}$

The mean orbital diameter expressed in astronomical units is 2.860 AU.

SC 3.

Given

$r = 5.203$ AU

Required

the orbital period of Jupiter (T)

Analysis and Solution

Jupiter orbits the Sun, so the constant $K = 1.0\ \text{y}^2/\text{AU}^3$.

$$K = \frac{T^2}{r^3}$$

$$T = \sqrt{Kr^3}$$

$$T = \sqrt{\left(1.0 \frac{\text{y}^2}{\text{AU}^3}\right)(5.203\ \text{AU})^3}$$

$$T = 11.87\ \text{y}$$

Paraphrase

The orbital period of Jupiter is 11.87 years.

SC 4.

Given

$r_T = 1.22 \times 10^9$ m
$r_D = 3.774 \times 10^8$ m
$T_D = 2.74$ d

Required

the orbital period of Titan (T_T)

Analysis and Solution

Both planets are orbiting Jupiter, so they will have the same constant for Kepler's third law. Equate the two ratios for Kepler's third law, and solve for the period of Titan.

$$\frac{T_T^{\ 2}}{r_T^{\ 3}} = \frac{T_D^{\ 2}}{r_D^{\ 3}}$$

$$T_T^{\ 2} = \frac{T_D^{\ 2} r_T^{\ 3}}{r_D^{\ 3}}$$

$$T_T = \sqrt{\frac{T_D^{\ 2} r_T^{\ 3}}{r_D^{\ 3}}}$$

$$T = \sqrt{\frac{(2.74 \text{ d})^2 \left(1.22 \times 10^9 \text{ m}\right)^3}{\left(3.774 \times 10^8 \text{ m}\right)^3}}$$

$$T = 15.9 \text{ d}$$

Paraphrase

The orbital period of Titan is 15.9 d.

Lesson 4

SC 1.

Given

$r = 6.97 \times 10^6$ m
$G = 6.67 \times 10^{-11}$ N·m²/kg²
$m_E = 5.98 \times 10^{24}$ kg

Required

the acceleration of the satellite (g)

Analysis and Solution

The acceleration of the satellite is due to the force of gravity from Earth.

$$
\begin{aligned}
g &= \frac{Gm_{\mathrm{E}}}{r^2} \\
&= \frac{\left(6.67\times10^{-11}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)\left(5.98\times10^{24}\ \mathrm{kg}\right)}{\left(6.97\times10^{6}\ \mathrm{m}\right)^2} \\
&= 8.19\ \mathrm{m/s^2}
\end{aligned}
$$

Paraphrase

The acceleration of the satellite is $8.19\ \mathrm{m/s^2}$.

SC 2. The magnitude of the acceleration vector decreases as it gets farther from Earth and increases as it gets nearer. The direction is always towards the centre of Earth.

SC 3. There is no noticeable change in the direction of the force and very little change in its magnitude.

SC 4.

Given

$r = 4.50 \times 10^{12}$ m $\quad m_{\mathrm{S}} = 1.99 \times 10^{30}$ kg

Required

the orbital speed of Neptune (v)

Analysis and Solution

Use the equation just derived to calculate the speed.

$$v = \sqrt{\frac{Gm_2}{r}}$$

$$v = \sqrt{\frac{\left(6.67\times10^{-11}\ \frac{\mathrm{N\bullet m^2}}{\mathrm{kg^2}}\right)\left(1.99\times10^{30}\ \mathrm{kg}\right)}{\left(4.50\times10^{12}\ \mathrm{m}\right)}}$$

$$v = 5.43\times10^{3}\ \mathrm{m/s}$$

Paraphrase

The orbital speed of Neptune is 5.43×10^3 m/s.

SC 5.

Given

$h = 359.2$ km $\quad r_E = 6.37 \times 10^6$ m $\quad m_E = 5.98 \times 10^{24}$ kg

Required

the orbital speed of the International Space Station (v)

Analysis and Solution

Convert the height to metres, and add it to the radius of Earth to get the radius of orbit. Then calculate the speed.

$$359.2\ \text{km} \times \frac{1000\ \text{m}}{1\ \text{km}} = 3.592 \times 10^5\ \text{m}$$

$$\begin{aligned} r &= r_E + h \\ &= (6.37 \times 10^6\ \text{m}) + (3.592 \times 10^5\ \text{m}) \\ &= 6.7292 \times 10^6\ \text{m} \end{aligned}$$

$$v = \sqrt{\frac{Gm_2}{r}}$$

$$v = \sqrt{\frac{\left(6.67 \times 10^{-11}\ \frac{\text{N} \bullet \text{m}^2}{\text{kg}^2}\right)\left(5.98 \times 10^{24}\ \text{kg}\right)}{\left(6.7292 \times 10^6\ \text{m}\right)}}$$

$$v = 7.70 \times 10^3\ \text{m/s}$$

Paraphrase

The orbital speed of the International Space Station is 7.70×10^3 m/s.

SC 6.

Given

$m_V = 4.87 \times 10^{24}$ kg
$r = 1.08 \times 10^{11}$ m
$T = 224.7$ d

Required

the mass of the Sun (m_S)

Analysis and Solution

Convert the orbital period to seconds so the units will cancel properly. The inward (centripetal) force is the force of gravity. Equate the formula for centripetal force involving period to the force of gravity, and solve for the mass of the Sun.

$$224.7\ \text{d} \times \frac{24\ \text{h}}{1\ \text{d}} \times \frac{3600\ \text{s}}{1\ \text{h}} = 1.9414 \times 10^{7}\ \text{s}$$

$$\begin{aligned}
F_g &= F_c \\
\frac{Gm_V m_S}{r^2} &= \frac{4\pi^2 m_V r}{T^2} \\
m_S &= \frac{4\pi^2 r^3}{T^2 G} \\
m_S &= \frac{4\pi^2 \left(1.08 \times 10^{11}\ \text{m}\right)^3}{\left(1.9414 \times 10^{7}\ \text{s}\right)^2 \left(6.67 \times 10^{-11}\ \frac{\text{N} \bullet \text{m}^2}{\text{kg}^2}\right)} \\
m_S &= 1.98 \times 10^{30}\ \text{kg}
\end{aligned}$$

Paraphrase

The mass of the Sun from Venus' data is 1.98×10^{30} kg.